DOCUMENT RESUME

ED 286 767 SE 048 607

TITLE	New Teachers Mathematics Staff Development, Grades 3-6.
INSTITUTION	New York City Board of Education, Brooklyn, N.Y. Div. of Curriculum and Instruction.
PUB DATE	[87]
NOTE	245p.; For related documents, see SE 048 605-608.
AVAILABLE FROM	New York City Board of Education, Div. of Curriculum and Instruction, 131 Livingston St., Rm 613, Brooklyn, NY 11201 ($6.00).
PUB TYPE	Guides - Classroom Use - Guides (For Teachers) (052) -- Guides - Non-Classroom Use (055)
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	Classroom Techniques; Critical Thinking; Educational Testing; Elementary Education; *Elementary School Mathematics; Grade 3; Grade 4; Grade 5; Grade 6; Homework; Mathematics Curriculum; *Mathematics Instruction; Mathematics Teachers; Problem Solving; Questioning Techniques; *Staff Development; Student Evaluation; Teacher Education; *Teacher Workshops
IDENTIFIERS	*New York City Board of Education

ABSTRACT

The New Teachers Mathematics Staff Development Program for grades 3 through 6 is a combination of content and methodology. Teachers focus their attention on content, lesson planning, questioning, homework, and testing. This volume contains materials for eight workshop sessions. Sessions included are: (1) An Introduction to the New York City School System; (2) Managing the Mathematics Classroom; (3) The Curriculum; (4) Planning and Executing an Effective Mathematics Lesson; (5) Questioning in the Mathematics Classroom; (6) Developing Reasoning Skills, Critical Thinking, and Problem Solving Ability in the Mathematics Classroom; (7) Homework Planning and Review To Maximize Student Learning; and (8) The Teacher's Role in Student Assessment. Reading materials for each section contain background materials, outlines, teaching tips, activities, and evaluation materials; materials provided in each section vary. (RH)

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Staff Development for New Teachers

MATHEMATICS

Grades 3-6

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Office of Curriculum Development and Support
Division of Curriculum and Instruction

2

ED 286 767

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FOREWORD

By making the decision to join the ranks of Mathematics educators of New York City, you have become part of a highly respected family of colleagues with a long tradition of excellence and professional commitment. I welcome you into this circle and remind you of the readiness of the Division of Curriculum and Instruction to make your entry into your classroom as smooth as possible.

There are many people who stand ready to help you, including your fellow teachers, your department supervisor, your principal and the staff of your Superintendent's office. As an introductory orientation to the responsibilities and joys of teaching Mathematics, this Staff Development Program for New Teachers will provide you with a detailed overview of the concerns of Mathematics teachers -- one that I believe you will be referring to time and again, even when you are no longer a new teacher.

In a sense, we are always "new" teachers. There is something about this profession that renews us every semester: the challenge of new youngsters, the challenge of new curricula, and the challenge of new personal goals for achievement. It is rewarding for me to be participating in your introduction to this exciting world, and I extend my very warmest wishes for every success.

Sincerely,

Charlotte Frank

Charlotte Frank
Executive Director,
Division of Curriculum and Instruction

ACKNOWLEDGMENTS

The New Teacher Mathematics Staff Development in Grades 3 through 6 Mathematics was developed as a project of the Mathematics Unit of the Division of Curriculum and Instruction, Charlotte Frank, Executive Director.

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INTRODUCTION

The New Teachers Mathematics Staff Development for Grades 3 through 6 is a combination of content and methodology. It is important to have a knowledge of what to teach as well as how to teach. Teachers will focus their attention on content, lesson planning, questioning, homework and testing. Teachers must become proficient in these essential areas in order to deliver effective instruction.

As participants master these significant aspects of teaching, a central thread runs through these areas. Student verbalization allows students to become active participants in the process of teaching and learning. Research has shown that expressing one's thinking through verbalization brings thinking to the conscious level. Students who take a verbally active part in each lesson through discussing, questioning, explaining and justifying their solutions have allowed their thoughts to be formulated and polished through the additional sense of hearing. Students who are asked to verbalize what strategies they have used to solve a set of problems are more successful on a subsequent set of related problems than students who have not been asked to put their strategies into words.

It is essential that teachers totally involve students in all aspects of the learning process. The teacher must have students respond during class in full sentence answers so that they are reinforcing not only what is done to solve a problem but also when and how the method is to be applied. The teacher must have students come to the board, write on the board and verbalize the strategies for solutions to the homework, class problems and teacher-made tests. Mathematics instruction becomes most effective when students actively discuss what they are learning.

Through verbalization, students see mathematics as a close knit system of understandable ideas, principles and processes. Students are more likely to retain mathematical learning over time if meaningful discussion activities comprise more than half of their mathematics instruction. Teachers must aim for a regular and consistent involvement of students through verbalization in order to assure meaningful instruction in mathematics. The crucial relationship between student verbalization and understanding is essential to insure student retention.

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NEW TEACHER STAFF DEVELOPMENT

Grades 3-6

WORKSHOP OUTLINE

MATHEMATICS

Session Title: An Introduction to the New York City School System.

The teacher will be able to:

1. Demonstrate an understanding of school policies, procedures, and regulations.
2. Identify records, reports, and forms used in the elementary school.
3. Organize a classroom for instruction.
4. Identify materials and supplies needed for the first week of school.
5. Explain the roles and responsibilities of the elementary school teacher.
6. Demonstrate an understanding of high school/local community school district organization.

Session Title: Classroom Management.

The teacher will be able to:

1. Identify the basic routines necessary to manage the class.
2. Develop classroom rules and routines and teach these to students.
3. Establish techniques for dealing with discipline problems.
4. Identify appropriate strategies for handling students with special needs.
5. Participate in simulations reflecting classroom management problems.
6. Develop strategies for improving student attendance.
7. Describe strategies that may be used to reduce mathematics anxiety.

Session Title: Effective Mathematics Instruction

The teacher will be able to:

1. Utilize the New York City Mathematics Scope and Sequence and appropriate strategies.

2. Identify the significant aspects of a mathematics lesson, and the resources that may be used in planning a mathematics lesson.

3. Explain strategies that may be used in executing a lesson to increase student retention of mathematical concepts and facts.

4. Explain how to plan homework using a cyclic approach to maximize student learning of mathematical concepts and facts, and to describe methods of reviewing students' homework.

5. Identify problem-solving strategies that can be taught to students to help them become better problem solvers.

6. Describe the steps to be taken in planning a mathematics test, in administering the test, in grading the test, in evaluating the test results, and in reviewing the test with students.

7. Describe procedures that may be used in organizing a classroom for mathematics instruction.

A Model for Adolescents

Teenagers want their teachers to be adults. Although they test their teacher's relationships with them, they expect their teachers to carry themselves as representatives of something larger than the immediate world of pop culture or the familiar world of their own friendships. In a world in which inspiration, guidance and positive values are often overshadowed by the bitter realities of existence, the teacher can be one of the most important persons in a youngster's life.

It is a misinterpretation of the teacher's role to see it as an opportunity to participate in adolescent activity in order to be accepted by adolescents. Respect for a teacher's adult point of view, developed through the teacher's own experience and education, is far more important than "chummy" recognition. Courts have described the teacher's position in certain situations as being in loco parentis -- "in place of the parent". The last thing a youngster needs from his teacher is one more person to whom to relate on the same levels on which he relates to his friends. What the youngster does need is a grown-up who will represent the highest values of society and education -- something to which to aspire, a good person to emulate.

The opportunities to express the special role of the teacher in the life of a youngster are many. Not all of them can be foreseen. It is often the spontaneous reaction of a teacher to a situation that leaves an indellible mark, positive or negative, in the memory of a youngster. Some opportunities, on the other hand, can be foreseen and methodically pursued. A teacher's appearance and dress can be designed to reflect his or her adult, professional position in society. A teacher's speech and choice of vocabulary can be made free of vulgarity. A teacher's personal department with students can be kept free of familiarity or intimacy, free of easily misunderstood verbal expressions and physical contact.

A teacher's reactions to the minute-by-minute unfolding of events during the school day can be monitored with respect to avoiding inappropriate anger, excessive fault-finding and criticism of people, devaluation of education or schooling, and disrespect for authority. When "something happens", youngsters will be looking at their teachers to see how to react. They can learn, from these examples of adult behavior, to emulate whatever model we present to them for emulation. In the larger world outside school, especially in the entertainment media, examples of hedonistic, violent,

vengeful, sensual and irresponsible behavior abound and are often presented in association with characters who are made to appear worthy of emulation. The teacher is one of society's few champions of another view of life: the reality of responsibility to one's fellow humans and to one's own potential, the abhorrence of violence, the recognition of higher pursuits than the mere satisfaction of appetites, the distinction between true happiness and simple pleasure. Overriding all of this is the teacher's commitment to the value of education as the vehicle for developing these values.

The Colleagueal Relationship

A school is an organic, dynamic set of relationships between the people who make it up: the teachers, the students, the parents, the supervisors and administrators. More than the building and its physical features and equipment, the school is this connective tissue between people. If -- heaven forbid! -- the building weren't there one Monday morning, the school, as embodied in these relationships, could still be alive and well.

Central to this fabric of interrelationships is the collegueal relationship between teachers. We are not a buildingful of isolated practicioners, each "doing our own thing". Our responsibilites extend to include the maintenance of supportive, respectful and sharing relationships with our fellow-teachers. For us to do our best job, we recognize our dependence on each other's special knowledge and experience. In this respect, teaching is different from other fields of activity in which competition and virtuosity are held in highest esteem. Our relationship to each other is closer to that of members of professions in which mutual respect, support and dependence is taken for granted and is regarded, in turn, with the highest respect by society in general.

Every teacher in the system is the teacher's colleague, since we are all working, through our own specialties, towards the same general principles. From this point of view, distinctions between teachers of different subject matter, teachers in different departments or in different schools, aren't nearly as important as the things that make us colleagues: our shared belief in the value of human potential and the central role that education plays in helping people to live up to it.

Teachers can help each other in many ways. More experienced teachers can help less experienced teachers to prepare lessons. Teachers can observe each other's teaching. Teachers can share copies of materials that they have prepared with each other. We can pay special attention to adhering to courses of study and calendars of lessons, since deviation from them leads to differences in the way students are prepared

and invites comparision between the work of different teachers of the same subject. We can make it a point to complete paper work accurately and one time, since failure to do so invariably means the creation of problems for colleagues who are waiting for it.

Everyday we can add to or subtract from this ideal relationship between colleagues. When we "put down" another teacher in speaking to students, when we elevate ourselves above other teachers, when we fail to cooperate with fellow teachers, we tear down this relationship. When we share with fellow teachers, when we come to their aid, when we speak of them in a way that elevates all teachers, we are building that relationship. Teaching, with a capital "T", benefits when students, parants and teachers observe this positive relationship in operation.

Professional Organizations

One of the best ways to become even more widely involved with colleagues is to join and be active in professional organizations. Through the publications of these organizaions, you can keep abreast of developments in the field of mathematics education as well as on-going discussion of issues that effect mathematics education. Through meetings and conferences that the organizations provide, teachers can interact with creative and enthusiastic colleagues in workshop experiences. As a teacher gains increased experience and special knowledge, he or she may become a speaker or workshop leader through an organization's invitation. The major professional organizations for mathematics educators are listed below.

ORGANIZATION	ADDRESS	CONTACT PERSON
National Council of Teachers of Mathematics (NCTM)	1906 Association Drive Reston, VA 22091	
Association of Teachers of Mathematics of New York City (ATMNYC)	P.O. Box 355 Thiells, NY 10984	Ms. A. Cacciola
Association of Teachers of Mathematics of New York State (AMTNYS)	R.D. Box 49 Coeymans Hollow, NY 12046	
Standing Committee of Mathematics	Mr. Noel Gurin Springfield Gardens H.S. 143-10 Springfield Blvd Springfield Gardens, NY 11413	Mr. Noel Gurin A.P. Math

Student Behavior

The teacher's greatest ally in getting students to exhibit their best behavior is an interesting, engaging activity for them to be involved with. Well-prepared lessons that stress student participation in a variety of ways (working with manipulatives at seats, working at the board, role-playing, making presentations, etc.) go a long way towards eliminating the boredom and inactivity that so often lead to disruptive student behavior. We distinguish between "busy work", that is, repetitive, time consuming work for no apparent purpose, and engaging work -- work that is perceived as interesting and important by students. We can do a lot to broadcast our own attitudes about particular bits of work, and students are quick to pick them up. When we repeatedly introduce exercises with the invitation "Now let's do another one" we are charactersing the work as an endless procession of inherently uninteresting exercises. When we preface new material with apologetic expressions such as "Now here's a hard one" or "There's a fancy word for this ..." we are prompting students to regard what is coming as something negative. When every lesson begins in exactly the same way, with a routine, unengaging "Do Now" on the board, we are setting up low expectations for the lesson to follow.

Of course, the most engaging of teachers is going to have instances of behavior problems. Often, these are expressions of problems that students are harboring and about which we are completely ignorant. Youngsters these days are walking around with burdens that often overshadow anything that school may have to offer by way of diversion. Many students live in one-parent homes. Others are living in homes that are about to become one-parent homes. Many students have financial responsibilities after school and/or before school that may render them tired and inattentive. Even youngsters who do not have problems of this magnitude are burdened by the normal but considerable problems of adolescence. Many of them are insecure, over-sensitive, and easily embarrassed. It is safe to assume that, in some way, they are all at least a little worried that they are not good enough and are trying desperately to prevent the world from finding this out. Little wonder that, as they make their way through a day in school, interacting with seven or more teachers and hundreds of fellow students, so many of them exhibit unacceptable behavior.

Each teacher can at least see to it that he or she provides no stimulus for triggering bad behavior. We avoid, for instance, remarking unfavorably (even in jest) about a youngster's appearance, because this is one of the surest ways of eliciting a bad response. We avoid public verbal exchanges with youngsters, since these often evoke the face-saving instinct that leads to escalated "one word led to another" situations. We avoid making emotional outbursts in anger, especially raising our voices, because these inspire like behavior in students.

On the other hand, we can do much to elicit the best from youngsters by communicating that we respect them and are sensitive to problems. Sincerely complimenting students for good work helps to create a positive reward that most students will want to work for. Inquiring about a student's health after an illness and providing for a reasonable way for making up missing work or exams can go a long way towards communicating the teacher's concerned decency and eliciting a like response. Giving students the benefit of the doubt often calls up their best cooperation.

Wherever possible, behavior incidents should be handled by the teacher, if not at the moment, then after class. Habitually resorting to such methods as expelling a student from class (which is illegal), sending a student to the department chairman (which is not generally appreciated) or to the dean's office communicates very clearly to students that you are incapable of handling situations yourself. These methods undermine the development of the teacher's own ability to deal with difficult situations.

With increased experience, the teacher learns to distinguish between run-of-the-mill incidents which he or she can handle effectively without help and special incidents which are indicative of very serious problems. In such cases we welcome the help that colleagues can give. Drug-related problems, problems that involve weapons, fighting, theft or other crime, problems that may stem from child-abuse situations, problems that may relate to undetected physical or psychological disorders -- all of these are situations in which the full range of school specialists should be enlisted. Consultation begins with the department head and may then involve the guidance counsellor, the SPARK coordinator, or outside agencies. Often what is manifested as poor behavior is really an undetected learning disability that is best brought to the attention of Special Education specialists.

The First Few Days

The first few days of a new school year, semester or cycle will seem like a movie run at three times the normal speed. You will wonder where the hours went and how anyone is expected to teach anything to anyone under conditions that may include any or all of the following: students dropped and added to your classes at the worst possible moments, textbooks running short when you least expect it, chalk, erasers, book receipts, delaney cards running out when you most need them, faculty and department conferences scheduled when you have the least amount of time for them, students approaching you with the discovery that they are in the wrong class, etc., etc. It is a trying time for teachers, but also an exciting one as you anticipate teaching new groups of youngsters and, perhaps, new subject matter. The object is to get through these first few days with as little unexpected difficulty as possible, and knowing what to anticipate can help you to avoid much of this. Here are some suggestions to follow before the first few days of teaching:

* Ask your supervisor for copies of the grading policy, the attendance and cutting policies.

* Familiarize yourself with the school schedule, noting the beginnings and endings of periods and the start and end of teacher time for the school day.

* Learn the procedure for calling in sick. Check with your supervisor as to the procedure for leaving materials he/she can use for your replacement when you are absent.

* Locate your mailbox, both in the general office and in the department office, if one has been assigned to you.

* Obtain the following supplies: chalk, eraser, book receipts, delaney cards, delaney book.

* Obtain copies of the courses of study for the classes that you will be teaching, including published curriculum guides if they exist for your courses.

* Find out the names of other teachers who will be teaching the same courses that you will be teaching.

* Find out if a grade leader has been designated for your courses and what your duties are with regard to the grade leader.

* Obtain keys to the rooms in which you will be teaching, the department office, if this is the policy in your school, and the teacher's bathroom.

* Inquire of your supervisor what the preferred procedures are for taking attendance, reporting cutting, filling out delaney cards, distributing textbooks, collecting homework, etc.

* Learn from your supervisor acceptable formats for lesson plans. Set up a lesson plan notebook and plan the first several lessons, leaving time for administrative matters.

* Become aware of specific goals related to your school and the mathematics department regarding the mathematics program (i.e., improving regents examination results, improving student attendance, improving reading and writing skills) and relate them to your class instruction.

* Familiarize yourself with the "chain of referral" in your school, should problems arise.

* Familiarize yourself with the physical layout of your school, noting the locations of main offices.

* Remember, above all else, that students don't need to know how disoriented you may be. They are expecting you to be a source of stability and reassurance. Especially if they are also new to the school, they are even more in need of these than you are.

Before things are too far under way, establish classroom routines which will facilitate the optimum use of instructional time. Inform students right from the start what will be expected of them in terms of work and classroom behavior. Remind them about attendance and lateness regulations. Explain to them what polices will be followed in formulating their grades, and make sure that whatever policies you adopt are consistent with department, school and High School Division policies. Whatever policies are being used, the data upon which grades will be based must be documented in writing, either in the form of notations on delaney cards, in a marking book or on forms of your own design. If class participation is an element to be considered in formulating grades, then class participation must somehow be recorded. It is embarrassing for the teacher and supervisors when a grade is questioned and no documentation can be provided to back it up.

Establish routines for collecting and returning homework papers and exam papers, and get into the habit of using students to do classroom chores like erasing the boards and distributing and collecting papers. Instruct students regarding the important opening few minutes of each lesson, including the manner of presenting homework at the chalkboard and working on an opening activity.

One of your primary goals in these routines is to make it possible for you to use as much time as possible for instruction. In a forty-minute period, each minute represents 2.5% of the available time. If excessive time is spent on reviewing homework, passing papers or calling the roll, little time will be left for teaching and learning. Develop quiet and efficient ways of checking attendance without using instruction time for this, waiting until students are busy at work at their seats. Learn how to postpone necessary student conferences until the end of the class instead of interrupting instruction to discuss a student's individual problems with attendance, lateness, etc. "Please see me after class" can be an important time-saver for you and your students. This also will avoid embarassing public confrontations.

Class Participation

Students have different ways of learning best, and different ways of expressing what they have learned. Some have no trouble writing what they know under pressure of time, and, therefore, do very well on written exams. Others have a great deal of difficulty doing well on written activities including exams, but are very verbal and articulate when they have the opportunity to speak about what they know. Classroom activities should be varied enough to allow all students to excel in expressing what they know in a variety of ways. If all students were held to a grading policy that allowed only for written performance, many able students would be doomed to failure. Thus, class participation is more than merely a way of enlivening and "socializing" the classroom, although it certainly does do these things. It is an important tool in the teacher's job of finding out what students know.

The way in which class participation develops in a class is an outgrowth of the teacher's questioning skills, especially with regard to his or her reaction to student responses to questions. This aspect of teaching requires the greatest skill, and one should, therefore, begin to work towards mastery in this area right from the start. How class discussion develops can either encourage or kill critical thinking, give confidence or rejection to students, facilitate or impede the development or problem solving skills, provide for or rule out the development of reasoning skills. Without going into a detailed account of questioning skills here, suffice it to recall that the most important word in the teacher's vocabulary is "why" and the greatest challenge is getting students to think more and say more.

The greatest sensitivity with regard to class participation is required when dealing with students for whom English is not the first language. Already self-conscious about expressing themselves in English, expressing themselves regarding the abstractions of mathematics poses a double threat. Yet, to leave them unchallenged to try to do so would be to do them a disservice, while forcing the issue to the point of embarrassment is also doing them no favor. The goal should be, as with all students, to take them from where they are to a level of expression that is more than they would have achieved without your intervention.

Conditions in your classroom may be very special with regard to this situation. You may have large numbers of students all with similar language backgrounds, in which case the preparation of mathematics glossaries, with student input, may be helpful in providing an on-going reference for recurring vocabulary words. On the other hand, your classroom may contain a great variety of language backgrounds, in which case you may want to confer with colleagues who can help with specific needs.

An apparent paradox of mathematics education is that students who may exhibit nearly total inability in English can be extremely proficient, even superior, in mathematics. Of course, this is no paradox. Simply imagine yourself transported to a country in which you are illiterate or barely literate and consider what effect this would have on your mathematical ability. The greatest effect would be on your ability to express your mathematical ability, not on your ability itself.

Fortunately, mathematics is a language unto itself, and the mathematics teacher can capitalize on this by providing students with the technical vocabulary they need to express the understandings which, in many cases, they already have. Mathematics, because of its universality, is often the one class in which students having difficulty with English can excel. It is everyone's second language.

The High School Division and the Community School Boards

It is important for a teacher to understand the position that his or her school occupies in the larger structure of the New York City school system. All too often teachers think in terms of "The Board" or "The System" without understanding who it is they really should be referring to.

Your High School

The New York City high schools are administered by the High School Division of the Board of Education. This is a centralized organization, with this one division setting policy for all the high schools in the city. Thus, all the various and many school policies regarding attendance, grading, curriculum, etc. must be designed so that they are consistent with High School Division policy.

To facilitate administration (there are some 111 high schools in New York City) the high schools are further subdivided geographically into five superintendencies, each with its own Superintendent of High Schools. These follow the borders of the five boroughs, except in the case of BASIS (Brooklyn and Staten Island Schools), which consists of the high schools of Staten Island and some from Brooklyn. The superintendent is the principal's direct supervisior. Become familiar with broad goals of the high school division related to mathematics instruction. Take advantage of staff development programs that are offered centrally and through your district superintendent. Your supervisor will be able to assist you with this.

Your Intermediate School:

If you are teaching in a junior high school or intermediate school, your school is part of a Community School District, of which there are 32 in the city. Each district has its own administrative staff, headed by a superintendent and staffed by a variety of specialists who perform many of the same functions for the elementary and intermediate schools in the district that the High School Division performs for the high schools of the city. This is a more de-centralized organization, with many of the policies determined at the community level. Ultimately, the Community School Boards are under the umbrella structure of the New York City Board of Education, and their schools participate in the full range of central activities, but in many areas they exercise a greater degree of autonomy than individual high schools do.

It is important to become familiar with the community school district's resources that are available for mathematics instruction. Make an appointment to meet the District Mathematics Coordinator. He or she is a resource person who can assist you be providing additional resources and supplies.

It will enhance your professional growth to participate at conferences with other mathematics teachers. This will allow you to stay current in the field of mathematics as well as in the area of effective instruction. Through your District Mathematics Coordinator you will be able to learn about new curriculum manuals to meet New York City and New York State guidelines as well as district or citywide staff development programs.

Situations for Discussion by New Teachers: Introductory Session

1. The teacher who occupies your room the period before you habitually keeps the class in beyond the end of the period, causing your class to start late each day. What do you do?

2. Between classes, a student passes you in the hall and verbally threatens you with physical harm. What do you do?

Consider the following interaction between a teacher and some students.

Teacher: Before we begin today's lesson, I want to remind you of the rule for participaton. Raise your hand before speaking.

Student A: (Without raising his hand.) You mean, if I have a question I need to raise my hand?

Teacher: That's correct.

Student B: (Without raising her hand.) What are we going to do today?

Teacher: I'll outline the lesson in a minute.

Student C: (Without raising his hand.) I thought you said we had to raise our hands?

Teacher: You're supposed to.

What are the consequences of the above?

Items to Avoid:

1. A preoccupation with one's authority may provoke rather than stiffle defiance and resistance to discipline. The teacher should not be concerned with her own prestige.

2. Refrain from nagging and scolding, since it may fortify the child's mistaken concept of how to get attention.

3. Do not ask a child to promise anything. Most children will promise to change in order to get out of an uncomfortable situation. It is a sheer waste of time.

4. Do not give rewards for good behavior. The child may then work only in order to get his reward and stop as soon as he has achieved his goal. What's more, this will only strengthen his belief that he must be paid every time he acts civil or makes a contribution.

5. Refrain from finding fault with the child. It may hurt his self-esteem and may discourage him.

6. Avoid double standards - one for yourself and another for the students. In a democratic atmosphere everybody must have equal rights. This includes the chewing of gum, swearing, tardiness, unnecessary visiting, and talking with members of the faculty in class when the children are working, sitting when the class pledges allegiance, checking papers or doing any kind of work that prevents the teacher from looking at the child when he is talking to her.

7. Do not use threats as a method to discipline the child. Although some children may become intimidated and conform for the moment, it has no lasting value since it does not change their basic attitudes.

8. Do not be vindictive; it only stirs up resentment and unfriendly feelings.

Items to Consider:

1. Because problem behavior is usually closely related to the child's faulty evaluation of his social position and how he must behave in order to have a place in the class group, the teacher's first concern must be to understand the purpose of his behavior. Only then will the teacher be in a position to plan more effectively for this child.

2. Give clear-cut directions for the expected action of the child. Wait until you have the attention of all class members before you proceed in giving directions.

3. Be more concerned with the future behavior of the child rather than with past behavior. Refrain from reminding the child what he/she used to be or do.

4. As soon as a child misbehaves and tends to threaten the general atmosphere in the class, give him the choice either to remain in his seat without disturbing the others or to leave the classroom if possible or go to the back of the room.

5. Build on the positive and minimize the negative. There is much good in every child, but if you look only for academic achievement, you may never find it.

6. Try to establish a relationship with the child built on trust and mutual respect.

7. Discuss the child's problem at a time when neither of you is emotionally charged, preferably in the regular class discussions.

8. Use natural consequences instead of traditional punishment. The consequence must bear a direct relationship to the behavior and must be understood by the child.

Items to Consider:
(continued)

9. Be consistent in your decisions. Do not change a decision arbitrarily just because it suits your purpose at the moment. Inconsistency confuses the child about what is expected of him at a certain procedure.

10. See Behavior in its proper perspective. In this way, you will avoid making a serious issue out of trivial incidents.

11. Establish cooperative planning for future goals and the solution of problems.

12. Let children assume greater responsibility for their own behavior and learning. They cannot learn this unless you plan for such learning. Teachers who are afraid to leave the room because of what might happen prevent the children from taking responsibility. Responsibility is taught by giving responsibility. Be prepared for children to act up at first. Such training takes time.

13. Use the class as a group to express disapproval when a child behaves in an antisocial manner.

14. Treat the child as your social equal.

15. Combine kindness with firmness. The child must always sense that you are his friend, but that you would not accept certain kinds of behavior.

16 At all times distinguish between the deed and the doer. This permits respect for the child, even when he does something wrong.

17. Guide the individual to assume independence and his own self-direction.

18. Set the limits from the beginning, but work toward mutual understanding, a sense of responsibility, and consideration for others.

19. Admit your mistakes - the children will respect your honesty. Nothing is as pathetic as a defeated authoritarian who does not want to admit her defeat.

20. Mean what you say, but keep your demands simple, and see that they are carried out.

21. Children look to you for help and guidance. Give them this security, but make cooperation and eventual self-control the goal.

22. Keep in mind your long-term goal: an independent, responsible adult.

23. Children need direction and guidance until they can learn to direct themselves.

24. Close an incident quickly, and revive good spirits. Let children know that mistakes are corrected and then forgotten.

25. Commend a child when his behavior in a situation shows improvement.

26. Work cooperatively with the children to develop a procedure for dealing with infractions of the rules.

27. "Do unto others as you would have them do unto you."

As teachers we need to exercise kindness but also firmness so that the children will know what to expect from us; thus mutual respect between students and teachers will be the result of the democratic educational process. In this way the child will be able to project the inner order that exists in him out into his environment and will be able to make order in his own way, find his own place, and develop self-discipline.

When the students understand what is expected of them, when they have been accepted and respected as equal partners, when the teachers no longer feel threatened, then and only then, will they be ready to move into the process of learning in its true sense.

"Maintaining Sanity in the Classroom" - Dreikurs, Grunwald, Pepper

DISCIPLINE

I. **Some general principles**

A. Teacher attitudes and understanding

1. Have a sincere regard and concern for students

2. Be sensitive to the needs of all students

3. Discern the ways in which children differ

II. **Preventive Techniques**

A. Ways of Maintaining an Orderly Classroom

1. Make a good beginning. i.e., when meeting a class for the first time, the teacher should explain his procedures, standards, and regulations, and the reasons for them. As a rule of thumb, it is better to be relatively strict with new groups, but the requirements and regulations set by the teacher should be reasonable. It is much less difficult to ease requirements than to stiffen them.

2. Maintain standards. Once the teacher has adopted certain standards and procedures, he should insist that they be observed. To make an exception is to invite another exception, and another, and another.

3. Do not begin a class until pupils are attentive. Until orderly conditions are present, effective learning cannot take place. Stop when there is noise/inattention.

4. Be thoroughly prepared every day. Thorough preparation assures the teacher of knowing, during every minute of the class period, exactly what he will do and how he will do it. Such a knowledge by adding to his self-confidence, enables him to unfold the day's activities with assurance. Students are unlikely to probe for weaknesses in the teacher if he appears to be a competent individual. In additional, thorough preparation keeps the work of the day moving along in such a way as to keep all students continuously active, leaving no time for mischief.

5. Vary your procedures. Variety adds spice to teaching, whereas the same routine day after day is flavorless. It is difficult to hold the attention of pupils for long periods of time with an inflexible method of teaching. Variety of procedures adds interest, and helps to maintain attention. A common mistake made by teachers is to overuse the lecture method. They explain, describe, or narrate to the students, while their pupils occupy themselves by taking notes -- a procedure which, if continued, is deadly to holding interest and attention.

6. Keep your students active and they will have no time for mischief. The teacher's lesson plan should include ways of keeping all students active at all times, through written work, oral work, or board work. When a topic is being explained or discussed, the teacher may call on a pupil to summarize or the pupil may be asked a question on the material. The entire class should be thinking about the teacher's question and each pupil's answer each time a question is asked.

7. Treat your pupils as individuals. Know your students by name as soon as possible. Even though it is well for a teacher to be friendly toward his students and to show an interest in them, he should also keep in mind that there should be a line of demarcation between pupils and their teachers beyond which neither should step. This is not to say that the teacher should feel superior to his pupils; it is to say, rather, that a teacher should maintain his dignity as one in authority or he will lose the respect of his pupils and be unable to control them. Try not to be influenced by the reputation a pupil has earned in other classes with other teachers. Take your students at face value.

8. Be fair. Although a teacher is sure to like some of his students better than others, he should never allow this feeling to enter into the evaluation of their progress. It is sometimes necessary to consider extenuating circumstances, or to sprinkle justice with charity, but this acceptable practice is altogether different from allowing feelings to influence judgment.

9. Admit mistakes

10. Have a sense of humor within limits.

III. Ways of Dealing with Disorderly Pupils

A. Handling group infractions or misbehavior.

1. Don't wait until a class is out of hand.

2. Focus on individual culprits rather than upon the class in general. Divert ringleaders by questions or assigned appropriate activities.

3. Keep your temper and your poise. Pupils may be testing you.

4. Avoid threats you cannot or will not carry out.

5. If difficulties continue, analyze possible causes: your manner or attitude; management of routines, teaching procedures, etc.

6. Seek help or advice from experienced colleagues, dean or supervisor.

B. Handling Individual Offenders in the Classroom

1. Minor infractions (Stop and look at student; talk to student quietly; change seat). In a private talk, the pupil is more likely to cooperate,

there being no need for him to resort to face-saving or attention-getting devices, as there would be in the presence of his classmates. The danger in changing a student's seat is that the source of trouble may merely be transferred from one room location to another, in which event the affected student may need to be seated close to the teacher. More often than not, however, the student whose seat has been changed, although perhaps tending to act up for a day or so to save face by temporarily defying authority, will soon settle down in the new location. And even if the result is unsatisfactory, at least the teacher will have experimented with a less offensive approach before adopting a severer one.

2. After a correction, treat the issue as closed.

3. Emphasize the behavior, not the pupil.

4. Don't argue; avoid open clashes.

5. Don't force an issue so pupil is made to lose face; don't humiliate.

6. Frequent questioning. A measure that sometimes produces quick results with a pupil who is talkative or inattentive is to direct questions to him more frequently than to other students. It will not take the student long to recognize that he is questioned more frequently when he is disorderly than when he is not, and usually he will begin to give more attention than he did previously.

7. Don't force a student to publicly apologize to the class. A public apology usually accomplishes nothing constructive from the viewpoint of teacher-pupil relationships, and the smoldering resentment the student carries within him certainly is not conducive to willing work on his part.

8. Other negative characteristics to be avoided in the meeting out of punishment is anger, frequent warnings, threats, scoldings, and sarcasm. The teacher should deliver the needed reprimand in a normal voice, with a minimum of emotion and in a non-judgmental way. Even though sarcasm admittedly is a function of time, person and place, a studied avoidance of the very appearance of it in the classroom is a safer approach.

C. Dealing with discipline problems that cannot be settled in a class.

1. Talk to offender in private. Determine causes of misbehavior and try to reach agreement or understanding.

2. Make the punishment fit the offense.

3. Leave offender with the feeling that he has a chance to redeem himself.

4. Use the resources at your disposal: check pupil's permanent record, get suggestions from teachers who know pupil, speak with pupil, analyze

your approaches, communicate with or see parent in accord with school procedures, channel his energies into useful approaches, keep an anecdotal record for chronic offenders, consult informally with supervisor, counselor or experienced colleague.

5. After exhausting your resources without results:

   a. Refer case to supervisor, dean or counselor in accordance with school regulations.

   b. Include on referral form the specific offense and measures you have taken to help the offender.

   c. Send additional material such as anecdotal record.

IV. <u>How can I get pupil's to work with me not because they have to but because they want to</u>?

A. Show an individual interest in your pupils -- their health, their problems, their achievements.

B. Establish class procedures and standards of behavior.

C. Trust pupils without being an easy mark. Take a pupil's word unless you have a good reason not to.

D. Give pupils a feeling of success through assignment of reasonable tasks and your personal encouragement.

E. Praise the class and individual pupils for good work, cooperation or improvement. Show that you appreciate their efforts and abilities.

F. Stress the positive -- what should be done rather than what should not be done.

G. Be fair and consistent. Don't accept infractions at one time and condem them at another time.

H. Show pupils the same respect and courtesy that you expect from them. Avoid sarcasm. Don't be arrogant or condescending.

I. Show your pupils that you like them and are concerned about them. They will respond in kind.

HOW SHOULD ONE HANDLE SPECIFIC TYPES OF DISCIPLINARY INFRACTIONS?

CONSTANT TALKING: Ask pupil questions so that he myst pay attention.
Check pupil's work.
Assign to activity such as chalkboard
Change his seat.

CALLING OUT OR INTERRUPTING:	Insist that pupils must raise their hands and then be called on to recite. Refuse to accept or give credit for an answer that is called out. Stress that other pupils should be given a chance to respond. Talk to pupil after class if necessary.
REFUSAL TO DO WORK OR OPEN DEFIANCE:	Make notation, then speak to pupil privately. Determine cause: work not understood, physical indisposition, emotional reaction, lack of motivation, etc. Suit follow-up action to cause. Use resources as needed: Checking pupil's record for his background and achievement, etc.
FOOLING AROUND OR HORSEPLAY:	Keep class working so that the offender or offenders are isolated. Insist calmly but firmly that nonsense be stopped. Check work and make notation in marking book or a Delaney card. If misbehavior stops quickly, continue lesson as if nothing has happened. If misbehavior continues or is repeated, see offenders after class. Follow up as needed.
IMPROPER LANGUAGE:	Indicate calmly that language is improper and uncalled for. Don' t take obsenity as personal affront but as offense against class and school. If remark is just one outburst of temper, keep your own record but try to adjust matter by talking to pupil in private. Don't insist on public apology, if pupil is repentant, his attitude will be evident to the class. If improper language is repeated or is part of a poor behavior pattern, send referral form to dean.
LEAVING ROOM WITHOUT PERMISSION:	Don't try to stop pupil by physical force. Send another pupil with note to inform dean. Mark pupil as cutting for time he is out, and follow school procedure for cutting. Continue your lesson. Follow up when pupil returns, using resources at your disposal. Suit your actions to the causes and the pupil's attitude.

ARGUMENT BETWEEN PUPILS:	Separate pupils quickly. Seat them in different parts of the room. If argument stops, see each pupil separately. Make sure argument has been settled. If argument threatens to continue after class or after school, notify dean's office. If argument continues in class, send one of the disputants out with another pupil, to the dean, with a note. If fight starts, send for assistance and try to break up fight with help of other pupils. Don't step in between the combatants unless they are younger or smaller children.
TEACHER OR PUPIL POSSESSIONS MISSING:	Give culprit chance to return article immediately without penaly, and possibly without detection. If article is not returned and you find culprit, send referral form at once. Theft, as distinct from mere mischief, is a serious offense. If article is not returned or discovered, send promptly for dean. Keep class in room until dean or security arrives.

Class Control, Class Discipline and Their Relation to Good Teaching

I. Causes of Trouble with Class Control and Discipline

A. Starting to teach before the entire class is in order and "under control."

B. Poor Teaching Techniques: direct correlation between teaching and class control and problems of discipline.

1. Weak and ineffective motivation; lack of interest in the lesson.
2. Weak or uncertain aim of lesson.
3. Poor questioning; failure to challenge or to stimulate pupils.
4. Inadequate use of the blackboard and other visual aids.
5. Not enough variety of procedure from day to day.

C. Teacher's personality uncertain and variable from day to day; not animated, dynamic or forceful, inconsistent enforcement of rules.

D. Teacher lacks adequate knowledge of the psychology of adolescents.

1. Teacher "sees" or "hears" too much.
2. Teacher loses temper and insults or humiliates pupils in front of the class.
3. Strictness not tempered by a sense of humor.
4. Teacher "not consistent" in day-to-day approach to class.

E. Teachers often direct attention to front of the room and part of the class, rather than directing himself to the entire class.

II. Constructive Suggestions for Better Class Control and Better Discipline

A. Make conscious effort to improve rapport; be firm but fair; don't humiliate or belittle students' efforts. Develop a sense of humor but do not attempt to compete with top-flight comedians; be on friendly terms with the students; make yourself available for post-class, brief discussions; don't "see" or "hear" everything; teach with spirit and animation; try to gain respect and good will of your pupils; do not teach "down" to the students.

B. Start teaching only when all pupils are seated, in order, and under your complete control; do not tolerate pupils coming late to class; have students remove gum and place in wrapper; permit no wearing of hats in class; permit no eating of food or drinking of soda in class; permit no student to "rest his head" on desk or "sleep" during your teaching recitation.

C. Careful and Well-Thought-Out Lesson Planning; provide for --

1. Stimulating motivation that will excite interest in the lesson and give a sense of importance.

2. 5 or 6 pivotal questions that will call for meaningful responses, provoke discussion, give opportunity for pupil curiousity.

3. Colorful illustrative material and other visual aids.

4. Pupil activity and live pupil participation.

D. Read some up-to-date books on adolescent psychology. Guard against offending pupils by personal remarks; respect the dignity of each individual pupil.

E. Direct your attention to the entire class; keep the ENTIRE class under your control every moment of the period. Draw pupils from every part of the room into active participation.

F. See to it that every day you dismiss the class. Keep teaching to the last moment.

III. Dealing With Cases of Discipline

A. First rule: don't make a threat you won't or can't carry through.

B. Avoid "cross the room" argument with a pupil.

C. Control yourself; ask pupil to see you after class - it is most important to avoid giving pupil an "audience situation" which encourages showing off and open defiance of the teacher; you must see the pupil privately.

D. Change seats of trouble-makers, without being too conspicuous about it.

E. In serious cases take up the issue with the Chairperson man and the Deans.

REDUCING MATHEMATICS ANXIETY

Learning anxieties, born of previous defeats and frustrations, impair the learning ability of many students. Of all these learning anxieties, math anxiety is the most widespread and most disabling. While math anxiety infects some average and even above average students it is epidemic and paralyzing among retarded students. Prescriptions that ignore it are unresponsive to an aspect of the diagnostic process that demands attention.

I. Causes of Math Anxiety

A. Previous personal experiences with parents, siblings, et al.

B. Sex-role stereotypes, social pressure.

C. The lingering effects of poor schooling and irregular attendance.

D. Attitude towards authority.

E. Student perception that math content is inflexible or the assumption that assigning symbols and making abstractions is depersonalized.

II. Symptoms of Math Anxiety

A. Sweaty palms; flushed cheeks.

B. Inability to hear or concentrate on directions.

C. Fear of asking questions, of revealing "ignorance".

D. Inattention and avoidance.

E. Withdrawal and inactivity.

F. Detachment which makes it impossible to function (math block).

III. Organizational Practices

A. Reviewing departmental class organizational practice.

1. Insuring proper class placement of students.

2. Limiting the ability range of students.

B. Designing varied grouping practices within the classroom: individual, small group, class.

IV. Creating a Responsive Environment

A. Teacher-student relationships.

1. Warmth and responsiveness, optimism and energy
2. Firmness without threats or criticism
3. Commitment to student growth
4. Overt encouragement and praise
5. Knowing students personally: using questionnaire and similar devices
6. Showing that math takes effort (even by the teacher)
7. Discussing math anxiety openly
8. Inviting students' reaction to the level of difficulty of work and of the text
9. Avoiding emphasis on covering a predetermined amount of material

B. Classroom Climate and Tone

1. Uncluttered appearance
2. Informal seating - relaxed atmosphere
3. Attractive decoration: plants, pictures, etc.
4. Varied and abundant materials
5. A quiet corner for concentrated efforts, mathematical browsing
6. Provision for display of students' work
7. Freedom to ask questions, to admit lack of knowledge

V. Instructional Procedures

A. Diagnosis and Prescriptions

1. Individual diagnosis
2. Interpreting student error patterns
3. Individual prescriptions

B. Performance Objectives

1. Teaching for understanding
2. Learning how to solve problems

   a. Focusing on realistic expectation
   b. Emphasizing students' mathematical capabilities
   c. Removing the pressure required for memorization by accepting the students' use of reference material, use of texts for finding formulas, etc.
   d. Stressing that the approach is more important than the answer

Dangers to avoid: Meaningless memorization, excessive drill and practice, labeling problems or math as easy, stress on speed, etc.

C. Motivation

1. Using current needs
2. Using possible careers

D. Development

1. Stressing students' discovering for themselves
2. Developing high but realistic expectations
3. Providing a variety of activities to meet individual learning styles
4. Clarification of language problems

   a. Technical words frequently misunderstood: ratio, fraction, least common denominator, etc.
   b. Common words frequently misunderstood: multiply, of, subtract, etc.
   c. Mathematical terms with multiple meanings: negative, minus, etc.
   d. Problems underlying symbols like -, X, $( )^2$

E. Materials of Instruction

1. Using practical real-life materials
2. Using varied materials for stimulation and to meet individual needs
3. Training students to use texts as reference
4. Avoiding "over-simplified" materials

F. Socialization

1. Encouraging verbal analysis of problems
2. Encouraging students to explain to one another
3. Forming small groups (2-6) for constructive interaction

VI. Setting Standards

1. Developing high but realistic expectations
2. Developing standards that are individual, not competitive
3. Insuring consistent recordkeeping
4. Providing feedback to students and pa ents on progress

VII. Taking Tests

1. Discussing math anxiety
2. Skimming the entire test
3. Starting with a problem students can solve easily
4. Encouraging students to defer more difficult problems and return to them after other problems are solved
5. Asking questions if the wording is vague or unclear
6. Searching for solution, not formulas
7. Training students to own personal learning style
8. Remembering that tests are not the last word in determining self-worth

Some DO's and DON'Ts — Meeting with the Parent

1. Try to arrange the conference so that there will be no interruptions.

2. The teacher's greeting should be warm and friendly, with a warm up topic to open the conversation.

3. Begin with a positive statement of the student's abilities. The parent is less defensive, and it conveys interest and liking for the student.

4. Share observations about the student, comparing responses at home and at school before making an interpretation or judgment.

5. Let any advice grow out of mutual discussion and a growing insight on the part of the parent into the reasons for the behavior.

6. Discover how the parent is thinking and feeling about the student. The teacher cannot understand the student's behavior until he/she knows the parent's attitude.

7. Listen to the parent's complaints or criticisms of the school, accepting them as evidence of interest or as suggestions to be considered.

8. If a parent gives his/her reason for the student's behavior, accept it, and, if possible, lead the discussion on to other possible causes. Behavior is a result of many causative factors.

9. Listen, and then listen some more. The teacher did not invite the parent to lecture but to obtain as well as give help. Encourage the parent to talk, and listen to what is said.

10. Try to close the conference with a constructive, hopeful statement.

11. It is easier to build a cooperative relationship if the teacher is not behind a desk. (Use a table if available.)

12. The educator should not argue with a parent. Arguing will arouse resentment.

13. The teacher should not try to push his/her thinking onto a parent. Conclusions and recommendations should be reached through a process of discussion and mutual thinking.

14. The teacher should not criticize harshly or negatively. Destructive criticism can be fatal to build a cooperative relationship in parent-teacher conferences since most parents cannot be objective about their children. The emotional involvement is too great.

15. The educator should not give direct advice unless it is requested, and never as the single successful solution.

16. Do not imply that the parent or home is to blame for the student's behavior.

17. Do not overlook influences other than the home on the student's life.

18. Avoid discussing other children in the school. Such comments direct attention from the problem at hand, encourage competitiveness, and often harm school and neighborhood relationships.

19. Do not use blanket words such as immaturity and insecurity.

20. Do not assume that the parents want help or advice. Such assumption usually brings resistance because it implies a form of criticism.

21. Do not criticize or blame past school experiences or teachers because such comments tend to destroy confidence in all education. Attention should be centered on present needs and on plans for the future, not on past mistakes.

Managing the Mathematics Classroom

Data Collection Sheet for Simulation of Statistics Lesson

Name (Last, First) _____ School _____

Eye color _____ Hair color _____ Height (inches) _____

Age _____ Weight _____

- How did you feel about being asked for your age and weight?
- For the purposes of a lesson on statistics, what other data could the instructor have ased for that would have served just as well?
- What implications does this simulation have for the classroom teacher?
- How can the teacher anticipate the effects of classroom interactions on the feelings of students?

Situations for Discussion by New Teachers: Classroom Management

1. A high-achieving student finished the regular class assignment ahead of schedule and is now sitting at this seat reading a book.

2. One student comes habitually late to class every day. When you confront the student, the student says that the Phys Ed teacher keeps the students late everyday. What do you do?

NUMBERS AND NUMERATION

Topic 01

0.01.01 Identification of Numbers Around Us

The student will be able to...

*identify number names in the environment

0.01.02 Spatial Relationships

The student will be able to...

*distinguish between in front of and in back of
*distinguish between up and down
*distinguish between above and below
* distinguish between on and off
* distinguish between over and under

0.01.03 Classification by Attributes

The student will be able to...

*order sets of objects from smallest to largest
*order sets of objects from largest to smallest
*distinguish likenesses and differences
*distinguish between same and different
*solve oral word problems using skills learned in this module

0.01.04 Groups

The student will be able to...

*recognize the number of objects in a group without counting

0.01.05 Comparison of sets

The student will be able to...

*distinguish between all and none
*compare two groups of objects to determine which group has more objects
*compare two groups of objects to determine which group has less objects
*compare two groups of objects to determine which group has more or less objects
*compare two groups of objects to determine if the number of objects in both groups are the same

0.01.06 Vocabulary

The student will be able to...

- *distinguish between not enough and extra
- *recognize enough by means of sharing
- *identify amounts which are the same and amounts which are different
- *distinguish betweeen more and less
- *distinguish between most and least
- *distinguish among many, as many as and fewest

0.01.07 Numerals one (1) through four (4)

The student will be able to...

- *associate the numerals 1 through 4 with a group (set, lot, bunch, heap, pile) having one through four objects
- *recognize and identify the symbols 1 through 4 as numerals for the numbers one through four
- *write the numerals 1 through 4
- *identify the number of elements in given groups (1 through 4) by counting

0.01.08 More identification of numbers around us

The student will be able to...

- *identify personal data

0.01.09 Spatial relationships- extended vocabulary

The student will be able to...

- *distinguish between top and bottom
- *distinguish between before and after
- *distinguish between inside and outside
- *distinguish between in and on
- *distinguish between left and right
- *distinguish between next to and between
- *distinguish among first, middle and last

0.01.10 Conservation of the number in a set

The student will be able to...

- *name sets of objects arranged in vertical, horizontal or random position
- *name the number in a set of objects in a vertical, horizontal, or random position
- *understand that counting objects in different positions does not change the number

0.01.11 Numerals five (5) through nine (9)

The student will be able to...

*associate the numerals 5 through 9 with a group (set, lot, heap, pile) having five through nine objects

* recognize and identify the symbols 5 through 9 as numerals for the numbers five through nine

*write the numerals 5 through 9

*identify the number of elements in a given group (5 through 9)

0.01.12 Numeral zero (0)

The student will be able to...

*associate the numeral 0 with a group(set, lot, bunch, heap, pile) having no objects

*recognize and identify the symbol "0" as a numeral for the number zero

*write the numeral 0

*identify the elements in a group of zero objects

0.01.13 Numeral 10

The student will be able to...

*associate the numeral 10 with a group (set, lot, bunch, heap, pile) having ten objects

*recognize and identify the symbol "10" as a numeral for the number ten

*write the numeral 10

*recite the numerals 1 through 10 in correct counting order

*identify the number of objects in a group of ten objects

0.01.14 Sequence

The student will be able to...

*pick the object or number that comes before a given object or number

*pick the object or number that comes after a given object or number

*pick the number that is between two objects or numbers

0.01.15 Ordinal numbers first through fourth

The student will be able to...

*identify the ordinal position of each object in a row through 4 objects

*match the ordinal numbers first through fourth to the cardinal numbers (numerals) one to four

*identify the numeral associated with a given ordinal number

*recite the ordinal names first through fourth

0.01.16 Construct and count using a number line

The student will be able to...

*construct a number line

*count forward and backward on the number line

*compare the cardinal number of a set of concrete objects by matching them with the numbers on a number line

0.01.17 Ordinal numbers first through tenth

The student will be able to...

*match the ordinal numbers first through tenth to the cardinal numbers one through ten

*recite the ordinal names first through tenth

0.01.18 Numerals eleven (11) through fifteen (15)

The student will be able to...

*associate the numerals 11 through 15 with a group (set, lot, bunch, heap, pile) having eleven through fifteen objects

*recognize and identify the symbols 11 through 15 as numerals for the numbers eleven through fifteen

*identify the elements in a given group

*recite the numerals 1 through 15 in correct counting order

1.01.01 Spatial relationships

The student will be able to...

*distinguish between up and down (A)

*distinguish between above and below (A)

*distinguish among in, on and off (A)

*distinguish between over and under (A)

*distinguish between top and bottom (A)

*distinguish between inside and outside (A)

1.01.02 Spacial relationships extended

The student will be able to...

*distinguish between in front of and behind (A)
*distinguish among before, after, between and next to (A)
*distinguish between left and right (A)
*distinguish among first, middle, and last (A)

1.01.03 Classification by attributes

The student will be able to...

*distinguish between same and different (A)
*distinguish among the concepts of large, larger, and largest (A)
*distinguish among the concepts of small, smaller, and smallest (A)
*distinguish between likenesses and differences (A)
*solve oral story problems using the skills learned in this module (A)

1.01.04 One-to-one and more than one corespondence

The student will be able to...

*pair objects from unlike sets by means of a one-to-one correspondence without counting (A)
*group objects from unlike sets by means of a two-to-one correspondence without counting (A)
*group objects from unlike sets by means of many-to-one correspondence without counting (A)

1.01.05 Sets

The student will be able to...

*identify a set as a collection or group of objects (B)
*identify the members of a set determined by a specific characteristic (B)
*identify the empty set (B)

1.01.06 Compare groups of 1 through 5 objects

The student will be able to...

- *compare groups (sets, bunches, lots, heaps, piles) of 1 through 5 objects by matching and showing which have the same number of objects (A)
- *compare groups of 1 through 5 objects by matching and showing which have more objects when given two groups of objects (A)
- *compare groups of 1 through 5 objects by matching and showing which have fewer objects, when given two groups of objects (A)
- *compare groups of 1 through 5 objects by matching and showing "more" and "fewer" when given two groups of objects (A)

1.01.07 Quantities

The student will be able to...

- *distinguish between all and none (A)
- *distinguish between most and least (A)
- *decide whether the groups (sets, lots, bunches, heaps, piles) have the same number of objects (as many as) (A)
- *distinguish between more than and less than (A
- *distinguish between not enough and extra (A)

1.01.08 Numerals one (1) through five (5)

The student will be able to...

- *associate the numerals 1 through 5 with a group (set, lot, bunch, heap, pile) having one through five objects (A)
- *recognize and identify the symbols "1" through "5" as numerals for the numbers one through five (A)
- *recite the numerals 1 through 5 in correct counting order (A)
- *write the numerals 1 through 5 (A)
- *identify the number of objects in a given group of 1 through 5 (A)
- *associate the number words "one" through "five" with the numerals 1 through 5 and with groups having one through five objects (A)
- *locate where you can find the numerals 1 through 5 in the home, school and/or neighborhood (A)

1.01.09 Numerals six (6) through nine (9)

The student will be able to...

- *identify and associate the numerals 6 through 9 with a group (set, lot, bunch, heap, pile) having six through nine objects (A)
- *associate the number words six through nine with the numerals 6 through 9 and with groups having six through nine objects (B)
- *locate where you can find the numerals 6 through 9 in the home, school and/or neighborhood (A)

1.01.10 Order numerals one (1) through nine (9)

The student will be able to...

- *identify and order the numerals and number words from 1 through 9 (A)
- *identify the numeral that comes one after a given numeral (1 through 9) (A)
- *identify the numeral that comes one before a given numeral (1 through 9) (A)
- *identify the counting number that comes between two given counting numbers (A)`

1.01.11 Ordinal numbers first through fifth

The student will be able to...

- *identify the ordinal position of each object in a row of 1 through 5 objects (B)
- *recite the ordinal names(B)
- *match the ordinal number words first through fifth to the cardinal numbers 1through 5 (B)
- *write the numeral form for the given ordinal number work (B)

1.01.12 The numeral zero (0)

The student will be able to...

- *associate the numeral 0 with a group (set, lot, bunch, heap, pile) having no objects (A)
- *recognize, identify and write the symbol 0 as a numeral for the number zero (A)
- *match the number word zero to the numeral 0 and to a group (set, lot, bunch, heap,pile) having no objects (B)

1.01.13 The numeral ten (10)

The student will be able to...

*identify and associate the numeral 10 with a group (set, lot, bunch, heap, pile) having ten objects (A)

*associate the number word ten with the numeral 10 and with a group (set, lot, bunch, heap, pile) having ten objects (B)

*recite, count and write the numerals 0 through 10 forward and backward in correct counting order (A)

1.01.14 Ordinal numbers through tenth

The student will be able to...

*identify the ordinal position of each object in a row, through 10 objects (A)

*match the ordinal number names first through tenth to the cardinal numbers 1through 10 (B)

1.01.15 Use the equal symbol (=) to compare groups from 0 through 10 objects

The student will be able to...

*compare groups with 0 through 10 objects having the same number of objects(A)

*compare equal groups with 0 through 10 objects using the relation symbol = (equal), (A)

*compare the numbers 0 through 10 using the equal relation symbol (=) (A)

1.01.16 Greater than (>) and less than (<) symbols

The student will be able to...

*identify the group having more objects (A)

*use the greater than relation symbol (>) to compare numerals 0 through 10 (B)

*identify the group having fewer objects (A)

*use the less than relation symbol(<) to compare numerals 0 through 10 (B)

1.01.17 Count and group by tens through fifty

The student will be able to...

*identify groups of 10 (A)

*order and write numbers 10 through 50 by tens (A)

1.01.18 Place value from 10 through 50

The student will be able to...

- *identify and write the place value of numerals 10 through 50 (A)
- *write the numerals 10 through 50 in expanded notation (A)

1.01.19 Order numbers from 10 through 50

The student will be able to...

- *identify the numeral that comes one after a given numeral (10 through 50) (B)
- *count and write by ones from 10 through 50 in correct counting order (A)
- *identify the numeral that comes one before a given numeral (10 through 50) (B)
- *count and write backwards by ones from 50 (B)
- *identify the numeral that comes between two given numerals (B)
- *solve story problems related to skills learned in this module (A)

1.01.20 Place value from 50 through 99

The student will be able to...

- *order and write numbers 50 through 90 by tens (A)
- *identify and write the place value of numerals from 51 through 99 (A)
- *write the numerals 51 through 99 in expanded notation (B)

1.01.21 Order numerals from 51 through 99

The student will be able to...

- *count and write by ones from 51 through 99 in correct counting order (A)
- *identify the numeral that comes one after a given numeral (51 through 99) (B)
- *identify the numeral that comes one before a given numeral (51 through 99) (B)
- *identify the numeral that comes between two given numerals (B)

1.01.22 Count by twos through 98

The student will be able to...

- *identify the numerals in order from 0 through 98, counting by twos (B)
- *write the numerals in counting order from 0 through 98, counting by twos (B)

1.01.23 Even and odd numbers through 99

The student will be able to...

*decide whether or not a group (set, pile, lot, heap, bunch) has an even number of objects (A)

*identify even numbers from 0 through 98 (B)

*identify numbers that are odd (not even) from 1 through 99 (B)

2.01.01 Count hundreds through ten hundred

The student will be able to...

*count forward and backward by hundreds through ten hundred (1000) (B)

*write the numbers from 100 through 1000 by hundreds (B)

*identify and write the number that comes between a given pair of hundreds through 1000 (A)

*identify and write the number that comes one hundred before and one hundred after a given number (A)

2.01.02 Read and write standard numerals (100 through 1000)

The student will be able to...

*count and write numbers forward and backward through 1000 without bridging the hundreds (B)

*identify and write the number that comes exactly between a given pair of numbers through 1000 (A)

*identify the number that comes one before and one after a given number through1000 (A)

*count forward and backward by ones bridging the hundreds (99 through 1000) (B)

2.01.03 Count by tens and fives to 1000

The student will be able to...

*count and order numbers 50 through 100 by tens (A)

*write numbers 50 through 100 by tens (A)

*count and order numbers 100 through 1000 by tens (A)

*write numbers 100 through 1000 by tens (A)

*count and order numbers 0 through 1000 by fives (A)

*write numbers 0 through 1000 by fives (A)

2.01.04 Relation symbols =, <, >

The student will be able to...

*compare the numbers 0 through 1000 using the relation symbol = (equal) (A)

*compare the numbers 0 through 1000 using the relation symbol > (greater than) (A)

*compare the numbers 0 through 1000 using the relation symbol <(less than) (A)

*compare numbers 0 through 1000 using the relation symbols =,<,> (A)

2.01.05 Even and odd numbers

The student will be able to...

*identify even numbers from 100 through 1000 (A)

*identify odd numbers (not even) from 101 through 999 (A)

*complete even and odd number sequences (A)

2.01.06 Round numbers through 99 to the nearest ten

The student will be able to...

*round a 2-digit number, having its ones digit less than 5, to the nearest ten (A)

*round a 2-digit number, having its ones digit more than 5, to the nearest ten (A)

*round a 2-digit number, having 5 in the ones place, to the nearest ten (A)

2.01.07 Place value through 999

The student will be able to...

*identify the place value of any digit in a numeral less than 1000 (A)

*identify the meaning of zero in the place value system (A)

*write a given numeral in expanded form (A)

*write a standard numeral when the expanded form is given (A)

2.01.08 Ordinal numbers eleventh through thirty-first

The student will be able to...

*identify the ordinal position of each object in a row through 20 objects (B)

*identify the ordinal position of each object in a row through 31 objects (B)

2.01.09 Identify and write Roman numerals I-XII

The student will be able to...

*identify the Roman numerals I, V, and X (C)

*identify and write Roman numerals from I through XII (1 through 12) (C)

2.01.10 Count forward and backward by patterns of three

The student will be able to...

*identify the numbers from 0 through 99 counting by threes (B)

*write and order the numbers from 0 through 99, counting by threes (B)

*count and write the numbers from 99 through 0 (in backward order) by threes (C)

2.01.11 Count forward and backward by patterns of four

The student will be able to...

*identify the numbers from 0 through 100 counting by fours (B)

*order and write the numbers from 0 through 100 counting by fours (B)

*order and write the numbers from 100 through 0 (in backward order) counting by fours (B)

3.01.01 Count by tens and hundreds through 1000

The student will be able to...

*count and write by ones forward and backward bridging the hundreds (99 through1000) (A)

*count and write by tens forward and backward within the hundreds (A)

*count and write by hundreds forward and backward to and from 1000 (A)

3.01.02 Round 3-digit numbers to the nearest ten/hundred

The student will be able to...

*estimate large numbers (B)

*round a 3-digit number, having its tens digit less than 5, to the nearest hundred (A)

*round a 3-digit number, having its tens digit more than 5, to the nearest hundred (A)

*round a 3-digit number, having 5 for its tens digit, to the nearest hundred (A)

*round a 3-digit number to the nearest ten (A)

3.01.03 Place value through 9,999

The student will be able to...

- *identify and write 4-digit numbers in expanded notation (A)
- *identify and write a 4-digit number in standard form given its expanded notation (A)
- *regroup hundreds as thousands (B)
- *regroup thousands as hundreds (B)
- *write 4-digit numbers with a comma between the thousands and hundreds places (A)

3.01.04 Order numbers through 9,999

The student will be able to...

- *order 3-digit numbers in both increasing and decreasing orders (A)
- *order 4-digit numbers in both increasing and decreasing orders (B)

3.01.05 Even and odd numbers

The student will be able to...

- *identify even and odd addition patterns of whole numbers (A)
- *identify even and odd subtraction patterns of whole numbers (A)

3.01.06 Count by twos, threes, fours, fives

The student will be able to...

- *count by twos forward and backward within the hundreds (A)
- *count by fives forward and backward within the hundreds (A)
- *count by threes forward and backward within the hundreds (A)
- *count by fours forward and backward within the hundreds (A)

3.01.07 Ordinal numbers through 500

The student will be able to...

- *use an ordinal number to name a position in a row up to 99 (A)
- *identify the ordinal position of each object in a row or series up to 500 objects (A)
- *write cardinal numbers with corresponding ordinal numbers (B)

3.01.08 Negative numbers

The student will be able to...

*identify the negative (-) sign for numbers and use negative numbers in real life situations (B)

*locate the points for the negative numbers on a number line (B)

3.01.09 Roman numerals

The student will be able to...

*identify and write Roman numerals from XIII through XXXIX (13 through 39) (B)

*solve word problems related to skills learned in this module (B)

4.01.01 Place value through thousands

The student will be able to...

*regroup hundreds as thousands (A)

*regroup thousands as hundreds (A)

*order 4-digit numbers in both increasing and decreasing orders (A)

4.01.02 Place value of 5- and 6-digit numbers

The student will be able to...

*use a place value chart to interpret 5- and 6-digit numbers (A)

*identify and write 5- and 6-digit numbers in expanded notation (A)

*identify and write 5- and 6-digit numbers in standard form given their expanded notation (A)

4.01.03 Round 4- and 5-digit numbers

The student will be able to...

*round 4- and 5-digit numbers to the nearest thousand (A)

*round 4- and 5-digit numbers to the nearest hundred (A)

4.01.04 Even and odd patterns with addition and subtraction

The student will be able to...

*define an even number as a number which is divisible by 2 or when divided by 2 leaves no remainder (A)

*define an odd number as a number which, when divided by 2 has a remainder of 1 (A)

*use an addition chart to determine even and odd sums (A)

*determine if differences of even and odd numbers are even or odd (A)

4.01.05 Prime numbers

The student will be able to...

*recognize that a prime number has only two factors, one and itself (A)

*list all the prime numbers through one hundred (A)

*identify a composite number as any whole number having more than two factors (A)

*rename a composite number as a product of its primes (A)

4.01.06 Roman system to symbol C

The student will be able to...

*identify the Roman symbols I, V, and X and their values (A)

*identify the Roman symbols L and C and their values (C)

*rename a Roman numeral as its Arabic equivalent (C)

*rename an Arabic number as its Roman equivalent (C)

4.01.07 Patterns in numeration

The student will be able to...

*find pattterns in a sequence of numbers and complete the sequence (A)

*find patterns for square numbers (B)

*find patterns for triangular numbers (B)

4.01.08 Place value through hundred million

The student will be able to...

*identify place value to millions (A)

*identify place value through hundred millions (B)

*identify and write 7-, 8-, and 9-digit numbers in expanded form (B)

*identify and write 7-, 8-, and 9-digit numbers in standard form when given the expanded form (B)

*translate a verbal phrase into a number and a number into a verbal phrase (B)

5.01.01 Place value through hundred thousands

The student will be able to...

*identify and write the place value of numbers through hundred thousands (A)

*compare two 6-digit numbers using the correct relation symbol (<,>) (A)

5.01.02 Round 6-digit numbers

The student will be able to...

*round 6-digit numbers to the nearest ten thousand (A)

*round 6-digit numbers to the nearest hundred thousand (A)

*solve word problems related to skills learned in this module (A)

5.01.03 Roman numerals to M

The student will be able to...

*identify the Roman symbol D (500) and use it with the symbols I,V,X,L and C to write Roman numerals (C)

*identify the Roman symbol M (1,000) and use it with the symbols I,V,X,L,C and D too write Roman numerals (C)

*rename a Roman numeral as its Arabic equivalent (C)

*rename an Arabic number as its Roman numeral equivalent (C)

5.01.04 Prime numbers

The student will be able to...

*identify a prime number as one having exactly two factors:1 and itself (B)

*identify a composite number as any whole number having more than two factors (B)

*rename a composite number as a product of its primes (B)

5.01.05 Exponents

The student will be able to...

*identify and define the exponent as the number written above the base to show how many times the base is used as a factor (B)

*rename a standard number in expanded notation using exponents (B)

5.01.06 Place value through hundred millions

The student will be able to...

*identify and write a 7-,8-, or 9-digit number in standard and expanded form (A)

*compare two 6-,7-,8-, or 9-digit numbers using the correct relation symbol (<,>) (B)

5.01.07 Base 5

The student will be able to...

*identify the basic symbols in base five (A)
*read a place value chart (A)
*find the base ten equivalents of a base five number (B)
*compare numbers in the base five with numbers in base ten (B)

5.01.08 Base - 12

The student will be able to...

*identify a base 12 number and rename it as a base 10 equivalent (B)
*identify the pattern of symbols in a base 12 (duo-decimal) system; ($T=10_{ten}$, $E=11_{ten}$) (B)
*rename a decimal number (base ten) as a base twelve number (B)

6.01.01 Read, write and understand large numbers

The student will be able to...

*identify and write numbers through 10-digits in expanded notation (A)
*identify and write the standard numeral given the expanded form of numbers through 10-digits (A)
*rename a standard numeral in expanded notation using exponents (A)
*compare two large numbers using the symbols <, >, = (A)
*solve word problems related to skills learned in this module (A)

6.01.02 Exponents

The student will be able to...

*identify and rename a numeral with an exponent as a numeral in standard form (A)
*identify and rename any non-zero number to the zero power as one (B)

6.01.03 Prime numbers

The student will be able to...

*identify a prime number as a whole number greater than one having only two factors (A)

*identify a composite number as any whole number having more than two factors (B)

*determine the prime factorization of any composite number (B)

*rename a composite number as a product of its primes (B)

6.01.04 Roman numerals

The student will be able to...

*identify the bar in the Roman numeral system as a symbol for multiplying the number represented without the bar by 1,000 (C)

*rename a Roman numeral (including bar) as its Arabic equivalent (C)

*rename an Arabic numeral as its Roman equivalent (C)

6.01.05 Base 2

The student will be able to...

*read a place value chart (B)

*find the base ten equivalent of a base two number (B)

*compare numbers in a base two with numbers in base ten (B)

7.01.01 Palindromes

The student will be able to...

*identify a palindrome (A)

*find a palindrome by repeated additions (B)

*classify a palindrome (B)

7.01.02 Divisibility rules: 2,5,10

The student will be able to...

*identify and use the divisibility rule for 2 (A)

*identify and use the divisibility rule for 5 (A)

*identify and use the divisibility rule for 10 (A)

7.01.03 Divisibility rules: 3,4,6,9

The student will be able to...

*identify and use the divisibility rule for 3 (A)

*identify and use the divisibility rule for 4 (A)

*identify and use the divisibility rule for 6 (A)

*identify and use the divisibility rule for 9 (A)

7.01.04 Triangular numbers, square pattern

The student will be able to...

- *identify and construct triangular numbers (A)
- *identify the patterns for triangular numbers (A)
- *construct a square pattern (A)
- *identify patterns for squares (A)

7.01.05 Pascal's triangle

The student will be able to...

- *construct Pascal's Triangle (A)
- *identify patterns in addition of rows (A)
- *identify patterns in Pascal's Triangle (A)

7.01.06 Fibonacci numbers

The student will be able to...

- *list the first 10 numbers in the Fibonacci sequence (B)
- *identify patterns with Fibonacci numbers (A)

7.01.07 Sets

The student will be able to...

- *use set notation to list the members of a set from a description of a set (A)
- *identify as a finite set a given set with a specific number of elements (A)
- *identify as an infinite set a given set with an unlimited number of elements (A)
- *determine whether or not two sets are equal or equivalent (A)
- *identify an empty set or null set as a given set with no elements (A)
- *state the definition of consecutive numbers, even numbers and odd numbers in word problems (A)

7.01.08 Relationships

The student will be able to...

*determine subsets of a given set (A)

*determine the number of subsets of a finite set of up to eight elements using the expression 2^n , where n is the number of elements in the set (B)

*represent two disjoint sets with a Venn diagram (A)

*represent a set and a subset with a Venn diagram (A)

*represent and list the union of two given sets on a Venn diagram (A)

*represent and list the intersection of two given sets on a Venn diagram (A)

*find the Least Common Multiple (LCM) of the elements of two given sets of multiples (A)

7.01.09 Prime and composite numbers

The student will be able to...

*determine whether a number is prime or composite (A)

*list and order the prime factors of a number (A)

*express prime factors in exponential notation (A)

7.01.10 Squares and square roots

The student will be able to...

*find the square of a number (A)

*list the squares of numbers (A)

*identify the positive square root of a number that is a perfect square (A)

*find the positive square root of a decimal fraction which is a perfect square (B)

*solve verbal problems related to skills learned in this module (A)

8.01.01 Sequences

The student will be able to...

*find a rule for a sequence of numbers (A)

*find additional terms for a given sequence (A)

8.01.02 Positive exponents

The student will be able to...

*write a standard numeral given a number expressed as a power of ten (A)

*write a standard numeral given a number in exponential form (A)

*write a standard numeral in exponential form (A)

8.01.03 Negative exponents

The student will be able to...

- *evaluate bases with negative and zero exponents (A)
- *expand decimals using positive and negative exponents (A)
- *recognize and identify a negative exponent as using the reciprocal (A)

8.01.04 Real numbers

The student will be able to...

- *define real numbers as the set of rational and irrational numbers (B)
- *match points on a number line with rational and irrational numbers (B)
- *order real numbers (B)

ADDITION OF WHOLE NUMBERS

Topic 02

0.02.01 Addition

The student will be able to...

- *recognize that addition is a union of sets
- *recognize that when two (non-empty) groups of objects are put together, the resulting group has more objects than either of the two original groups

0.02.02 Add one

The student will be able to...

- *add 1 object to 1 object
- *add 1 object to 2 objects
- *add 1 object to 3 objects
- *add 1 object to 0 objects

0.02.03 Plus (+) and equal (=) symbols

The student will be able to...

- *identify the plus (+) symbol
- *identify the equal (=) symbol
- *identify the plus (+) and equal (=) symbols in pictorial representations

0.02.04 Add two

The student will be able to...

- *add 2 objects to 1 through 3 objects
- *add 2 objects to 0 objects

0.02.05 Add three

The student will be able to...

- *add 3 objects to 1 through 4 objects
- *add 3 objects to 0 objects

0.02.06 Addition extention

The student will be able to... .

- *add 1 object to 4 through 9 objects
- *add 2 objects to 4 through 8 objects
- *add 3 objects to 5 through 7 objects

0.02.07 Add four

The student will be able to...

*add 4 objects to 1 through 4 objects
*add 4 objects to 0 objects
*add 4 objects to 5 and 6 objects

0.02.08 Add five through nine

The student will be able to...

*add 5 objects to 1 through 5 objects
*add 6 objects to 1 through 4 objects
*add 7 objects to 1 through 3 objects
*add 8 objects to 1 through 2 objects
*add 9 objects to 1 object
*add 5 through 9 objects to 0 objects

1.02.01 Identify the plus (+) symbol and the equal (=) symbol

The student will be able to...

*recognize that addition is the union of groups (A)
*recognize that when two groups of objects are added together the resulting group is greater than either of the two original groups (A)
*identify the plus (+) symbol (A)
*identify the equal (=) symbol (A)
*identify both the plus (+) and the equal (=) symbols, using a horizontal format (A)

1.02.02 Number sentences with the plus (+) symbol and the equal (=) symbol

The student will be able to...

*identify a frame as a place for writing a missing numeral in a number sentence (A)
*identify the plus (+) symbol in number sentences using vertical format (A)
*write the plus (+) symbol and the equal (=) symbol in number sentences, using both vertical and horizontal formats (A)

1.02.03 Commutative property with sums through five

The student will be able to...

- *add two numbers having sums of 2 through 5, in both vertical and horizontal formats (A)
- *identify the commutative property in addition with sums through 5 (A)
- *write addition facts for sums through five, in both vertical and horizontal formats (A)

1.02.04 Commutative property with sums through ten

The student will be able to...

- *write addition facts for the numbers six through ten, in vertical and horizontal formats (A)
- *identify the commutative property in addition with sums through ten (A)

1.02.05 Associative property for three addends

The student will be able to...

- *use addition to name the number of objects in the new set formed by joining three sets,using vertical and horizontal formats (A)
- *demonstrate that the order of grouping three addends does not change their sum (associative property) (A)
- *record repeated addition of three addends (A)

1.02.06 Sums through eighteen

The student will be able to...

- *add two 1- digit numbers having sums through 18 in horizontal and vertical formats (A)
- *use an addition table to find sums through eighteen (B)
- *identify simple patterns on the addition table (B)
- *solve story problems related to skills learned in this module (A)

1.02.07 Add 2-digit numbers

The student will be able to...

- *add a 2-digit number and a 1-digit number without exchange in horizontal and vertical formats (A)
- *add multiples of ten (A)
- *add two 2-digit numbers without exchange (A)
- *solve story problems related to skills learned in this module(A)

2.02.01 Addition of 2- and 3-digit numbers without regrouping (exchange)

The student will be able to...

- *add three or more 2-digit numbers without regrouping (exchange) and check (A)
- *add a 3-digit number and a 2-digit number without regrouping (exchange) and check (B)
- *add money with one addend through $9.99 without regrouping (exchange) and check(B)
- *solve story problems related to skills learned in this module (B)

2.02.02 Addition of 3-digit numbers without regrouping (exchange)

The student will be able to...

- *add two 3-digit numbers without regrouping (exchange) and check (B)
- *add three or more 3-digit numbers without regrouping (exchange) and check (B)
- *add money through $9.99 without regrouping (exchange) and check (B)
- *solve story problems related to skills learned in this module (B)

2.02.03 Addition of 2- and 3-digit numbers using expanded notation without regrouping (exchange)

The student will be able to...

- *add 2- digit numbers using expanded notation without regrouping (exchange) (A)
- *add 3- digit numbers using expanded notation without regrouping (exchange) (A)
- *add a 2- digit number and a 3- digit number using expanded notation without regrouping (exchange) (A)

2.02.04 Addition of a 2-digit number and a 1-digit number with regrouping (exchange) ones to tens

The student will be able to...

- *add a 2-digit number and a 1-digit number with regrouping (exchange) using expanded notation (B)
- *add a 2-digit number and a 1-digit number with regrouping (exchange) ones to tens and check (B)
- *solve story problems related to skills learned in this module (B)

2.02.05 Addition of 2-digit numbers with regrouping (exchange) ones to tens

The student will be able to...

- *add two 2-digit numbers with regrouping (exchange) using expanded notation (B)
- *add two 2-digit numbers with regrouping (exchange) ones to tens and check (B)
- *solve story problems related to skills learned in this module (B)

2.02.06 Commutative and associative properties of addition through 999

The student will be able to...

- *use the commutative property of addition with numbers through 999 (B)
- *use tables to discover patterns of the commutative property (B)
- *use the associative property of addition with numbers through 999 (C)

2.02.07 Addition of 2- and 3-digit numbers with regrouping (exchange) ones to tens

The student will be able to...

- *add a 3-digit number and a 2-digit number with regrouping (exchange) using expanded notation (B)
- * add a 3-digit number and a 2-digit number with regrouping (exchange) and check (B)
- *compute addition examples with regrouping (exchange) using money through $9.99 and check (B)
- *solve story problems related to skills learned in this module (B)

3.02.01 Commutative and associative properties of addition through 999

The student will be able to...

- *use the commutative property of addition with numbers through 999 (A)
- *use the associative property of addition with numbers through 999 (B)

3.02.02 Addition of 3-digit numbers without regrouping (exchange)

The student will be able to...

- *add two 3-digit numbers without regrouping (exchange) and check (A)
- *compute addition examples having sums to $9.99 without regrouping (exchange) and check (A)
- *solve word problems related to skills learned in this module (A)

3.02.03 Addition of 3-digit numbers with one regrouping (exchange) ones to tens

The student will be able to...

- *add two 3-digit numbers with one regrouping (exchange) ones to tens and check (A)
- *solve word problems related to skills learned in this module (A)

3.02.04 Addition of 2- and 3-digit numbers with one regrouping (exchange) tens to hundreds

The student will be able to...

- *add two 2-digit numbers with regrouping (exchange) tens to hundreds and check (A)
- *add two 3-digit numbers with regrouping (exchange) tens to hundreds and check (A)
- *solve word problems related to skills learned in this module (A)

3.02.05 Addition of 3-digit numbers with two regroupings (exchange) ones to tens, hundreds

The student will be able to...

- *add two or more 3-digit numbers with two regroupings (exchanges), ones for tens, tens for hundreds and check (A)
- *compute addition examples having sums to $9.99 with two regroupings (exchanges) and check (B)
- *solve word problems related to skills learned in this module (A)

3.02.06 Addition of 4-digit numbers

The student will be able to...

- *add two 4-digit numbers without regrouping (exchange) and check (B)
- *add two 4-digit numbers with one regrouping (exchange) and check (B)
- *add two 4-digit numbers with two regroupings (exchanges) and check (B)
- *add two 4-digit numbers with three regroupings (exchanges) and check (B)
- *rewrite a 4-digit horizontal addition example in vertical format with regrouping (exchange), add and check (B)
- *solve word problems related to skills learned in this module (B)

4.02.01 Addition of numbers through 4-digits

The student will be able to...

- *estimate sums by rounding addends to the largest place (A)
- *add 2 or more addends in vertical or horizontal format with or without regrouping (exchange) and check (A)
- *solve word problems related to skills learned in this module (A)

4.02.02 Addition of 5-digit numbers with or without regrouping (exchange) in one or more places

The student will be able to...

- *add two or more 5-digit numbers and check (B)
- *solve word problems related to skills learned in this module (B)

4.02.03 Associative property of addition

The student will be able to...

- *use the associative property of addition (A)
- *solve word problems related to skills learned in this module (A)

4.02.04 Addition of 5- and 6-digit numbers with or without regrouping in one or more places

The student will be able to...

- *add two or more 5-digit numbers and check (A)
- *add two or more 6-digit numbers and check (A)
- *solve word problems related to skills learned in this module (A)

5.02.01 Addition of two addends through 6-digit numbers

The student will be able to...

- *add two 6-digit numbers with regrouping in any column (including money) and check (A)
- *estimate by rounding addends to the largest possible place (B)
- *solve word problems related to skills learned in this module (A)

5.02.02 Addition -three or more numbers through 999,999

The student will be able to...

- *estimate by rounding addends to the largest possible place (B)
- *add three or more numbers and check (A)
- *solve word problems related to skills learned in this module (A)

6.02.01 Addition of two or more numbers through 9-digits

The student will be able to...

- *add three or more numbers through hundred million with regrouping and check(A)
- *estimate by rounding addends to the largest possible place (A)
- *solve word problems related to skills learned in this module (A)

7.02.01 Addition of whole numbers

The student will be able to...

- *compute the sums of whole numbers and check (A)
- *estimate by rounding addends to the largest possible place (A)
- *solve verbal problems related to skills learned in this module (A)

SUBTRACTION OF WHOLE NUMBERS

Topic 03

0.03.01 Subtraction

The student will be able to...

- *recognize that when an object(s) is subtracted from a group, the resulting group is smaller than the original
- *solve oral problems related to subtraction skills using concrete materials

1.03.01 Concept of subtraction

The student will be able to...

- *recognize that subtraction is the removal of members from a group (A)
- *recognize that when a member is subtracted from a group, the resulting group is smaller than the original group (A)

1.03.02 The minus (-) symbol

The student will be able to...

- *identify and write the minus (-) symbol (A)
- *distinguish between the plus (+) and minus (-) symbols (A)
- *demonstrate that subtract, minus and take away are synonymous (B)
- *identify the minus (-) symbol in a number sentence using both vertical and horizontal formats (A)
- *write the minus (-) symbol in a number sentence, using both vertical and horizontal formats (A)

1.03.03 Subtraction facts through five

The student will be able to...

- *subtract numbers to obtain differences from 1 through 5, in both horizontal and vertical formats (A)
- *write a subtraction sentence using both the horizontal and vertical formats for minuends through 5 (A)

1.03.04 Number families through five

The student will be able to...

*identify related addition and subtraction facts through five (A)

*choose operation symbols for facts having sums and differences through 5 (B)

1.03.05 Subtraction facts through ten

The student will be able to...

*subtract numbers to obtain differences from 1 through 10 (A)

*write a subtraction sentence using horizontal and vertical formats for minuends through 10 (A)

1.03.06 Number families through ten

The student will be able to...

*identify related addition and subtraction facts through ten (A)

*choose operation symbols for facts having sums and differences through 10 (A)

1.03.07 Subtract a 1-digit number from 11 through 18

The student will be able to...

*subtract a 1-digit number from 11 through 18 where the difference will be 2 through 9 (B)

*identify related addition and subtraction facts for each number family through18 (A)

*check subtraction problems by using adddition (A)

*solve story problems related to skills learned in this module (A)

1.03.08 Subtract a 1-digit number from a 2-digit number without regrouping (exchange)

The student will be able to...

*subtract a 1-digit number from a 2-digit number without regrouping (exchange) (A)

*check subtraction problems by using addition (A)

*sove story problems related to skills learned in this module (A)

1.03.09 Subtraction of ten(s)

The student will be able to...

*subtract 10 from multiples of 10 through 90 (A)
*subtract multiples of 10 through 90 (A)
*subtract 10 from any 2-digit number through 99 (A)
*subtract multiples of 10 from any 2-digit number through 99 (A)

1.03.10 Subtract a 2-digit number from a 2-digit number without regrouping (exchange)

The student will be able to...

*subtract a 2-digit number from a 2-digit number without regrouping (exchange) (A)
*check subtraction problems by using addition (A)
*solve story problems related to skills learned in this module (A)

2.03.01 Subtraction of a 2-digit number from a 3-digit number without regrouping (exchange)

The student will be able to...

*subtract a 2-digit number from a 3-digit number without regrouping (exchange) and check (B)
*subtract money using a 3-digit minuend and a 2-digit subtrahend and check (B)
*solve story problems related to skills learned in this module (B)

2.03.02 Subtraction of a 3-digit number from a 3-digit number without regrouping (exchange)

The student will be able to...

*subtract two 3-digit numbers without regrouping (exchange) and check (B)
*subtract money through $9.99 without regrouping (exchange) and check (B)
*solve story problems related to skills learned in this module (B)

2.03.03 Subtraction of 2-digit and 3-digit numbers using expanded notation without regrouping(exchange)

The student will be able to...

- *subtract 2-digit numbers using expanded notation without regrouping (exchange) (A)
- *subtract a 2-digit number from a 3-digit number using expanded notation without regrouping (exchange) (B)
- *subtract a 3-digit number from a 3-digit number using expanded notation without regrouping (exchange) (B)

2.03.04 Subtraction of a 1-digit number from a 2-digit number with regrouping (exchange)

The student will be able to...

- *subtract a 1-digit number from a 2-digit number with regrouping (exchange) using expanded notation (B)
- *subtract a 1-digit number from a 2-digit number with regrouping (exchange) tens for ones and check (B)
- *subtract money through 99c with regrouping (exchange) tens for ones and check (B)
- *solve story problems related to skills learned in this module (B)

2.03.05 Subtraction of 2-digit numbers with regrouping (exchange) tens to ones

The student will be able to...

- *subtract a 2-digit number from a 2-digit number with regrouping (exchange) using expanded notation (B)
- *subtract a 2-digit number from a 2-digit number with regrouping (exchange) tens to ones and check (B)
- *subtract money through 99c and check (B)
- *solve story problems related to skills learned in this module (B)

2.03.06 Properties of subtraction

The student will be able to...

- *recognize that the commutative property does not apply to subtraction (B)
- *use a table to discover the inverse relationship between addition and subtraction (B)

3.03.01 Subtraction of 3-digit numbers without regrouping (exchange)

The student will be able to...

- *subtract two 3-digit numbers without regrouping (exchange) and check (A)
- *subtract money through $9.99 without regrouping (exchange) and check (A)
- *solve word problems related to skills learned in this module (A)

3.03.02 Subtraction of 2-digit numbers with regrouping (exchange) tens to ones

The student will be able to...

- *subtract a 2-digit number from a 2-digit number with regrouping (exchange) using expanded notation (A)
- *subtract a 2-digit number from a 2-digit number with regrouping (exchange) tens to ones check (A)
- *subtract money through 99c and check (A)
- *solve word problems related to skills learned in this module (A)

3.03.03 Subtraction of 3-digit numbers with regrouping (exchange) tens to ones

The student will be able to...

- *subtract a 1-digit number from a 3-digit number with one regrouping (exchange) tens to ones and check (A)
- *subtract a 2-digit number from a 3-digit number with one regrouping (exchange) tens to ones and check (A)
- *subtract a 3-digit number from a 3-digit number with one regrouping (exchange) tens to ones and check (A)
- *solve word problems related to skills learned in this module (B)

3.03.04 Subtraction of 3-digit numbers with one regrouping (exchange) hundreds to tens

The student will be able to...

- *subtract 3-digit numbers with one regrouping (exchange) hundreds to tens and check (A)
- *compute subtraction examples with one regrouping (exchange) having minuends to $9.99 and check (B)
- *solve word problems related to skills learned in this module (A)

3.03.05 Subtraction of 3-digit numbers with two regroupings (exchanges) tens to ones and hundreds to tens

The student will be able to...

- *subtract 3-digit numbers with two regroupings (exchanges), tens to ones, hundreds to tens and check (A)
- *compute subtraction examples with two regroupings (exchanges) having minuends to $9.99 and check (B)
- *solve word problems related to skills learned in this module (A)

3.03.06 Subtraction of 4-digit numbers

The student will be able to...

- *subtract two 4-digit numbers without regrouping (exchange) and check (B)
- *subtract two 4-digit numbers with one regrouping (exchange) and check (B)
- *subtract two 4-digit numbers with two regroupings (exchanges) and check (B)
- *subtract two 4-digit numbers with three regroupings (exchanges) and check (B)
- *compute subtraction examples having minuends with multiples of 100 or 1000 and check (B)
- *solve word problems related to skills learned in this module (B)

4.03.01 Subtraction of 4-digit numbers

The student will be able to...

- *estimate differences by rounding minuends and subtrahends to the nearest thousand (A)
- *subtract two 4-digit numbers without regrouping (exchange) and check (A)
- *subtract two 4-digit numbers with one or more regroupings (exchanges) and check (A)
- *compute subtraction examples having minuends with multiples of 100 or 1000 and check (A)
- *solve word problems related to skills learned in this module (A)

4.03.02 Subtraction of 5-digit numbers with or without regrouping (exchange) in one or more place

The student will be able to...

- *subtract a 2-, 3-, 4- and 5-digit number from a 5-digit number and check (B)
- *subtract a 2-, 3-, 4- and 5-digit number from 10,000 and multiples of 10,000 and check (B)
- *solve word problems related to skills learned in this module (B)

4.03.03 Subtraction of 5- and 6-digit numbers with or without regrouping in one or more places

The student will be able to...

- *subtract a 2-, 3-, 4-, 5 or 6-digit number and check (A)
- *subtract a 2-, 3-, 4- and 5-digit number from 100,000 and multiples of100,000 and check (A)
- *solve word problems related to skills learned to skills learned in this module (A)

5.03.01 Subtraction through 6-digit numbers

The student will be able to...

- *subtract a 6-digit number from a 6-digit number with regrouping in any column and check (A)
- *estimate differences by rounding minuends and subtrahends (less than 1 million) to the nearest 100,000 (B)
- *solve word problems related to skills learned in this module (B)

5.03.02 Subtraction through 6-digit numbers

The student will be able to...

- *estimate differences by rounding minuends and subtrahends (less than 1 million) to nearest 100,000 (A)
- *subtract 6-digit numbers and check (A)
- *solve word problems related to skills learned in this module (A)

6.03.01 Subtraction of large numbers

The student will be able to...

- *subtract large numbers with exchange in any column and check (A)
- *solve word problems related to skills learned in this module (A)

7.03.01 Subtraction of large numbers

The student will be able to...

- *subtract large numbers and check (A)
- *estimate differences by rounding to the largest possible place (A)
- *solve verbal problems related to skills learned in this module (A)

MULTIPLICATION OF WHOLE NUMBERS

Topic 04

2.04.01 Multiplication facts for twos through 10

The student will be able to...

- *show groups of two through 5 groups of two (A)
- *count by twos through 10 (A)
- *demonstrate that multiplication is the process of repeated addition of like addends (A)
- *use a number line to demonstrate counting by twos through 10 (A)
- *recognize x as the symbol for multiplication (A)

2.04.02 Multiplication facts for fives through 25

The student will be able to...

- *show groups of five through 25 (A)
- *count by fives through 25 (A)
- *use a number line to demonstrate counting by fives
- *demonstrate that multiplication is the process of repeated addition of like addends (A)

2.04.03 Multiplication facts for threes through 15

The student will be able to...

*show groups of three through 5 groups of three (A)
*count by threes through 15 (A)
*use a number line to demonstrate counting by threes (A)
*demonstrate that multiplication is the process of repeated addition of like addends (A)

2.04.04 Multiplication facts for fours through 20

The student will be able to...

*show groups of four through 5 groups of four (A)
*count by fours through 20 (A)
*use a number line to demonstrate counting by fours through 20 (A)
*demonstrate that multiplication is the process of repeated addition of like addends (A)

2.04.05 Commutative property of multiplication

The student will be able to...

*apply the commutative property of multiplication to the two times table through 5 groups of two (A)
*apply the commutative property of multiplication to the three times table through 5 groups of three (A)
*apply the commutative property of multiplication to the four times table through 5 groups of four (A)

3.04.01 Multiplication facts

The student will be able to...

*write multiplication facts for numbers 1 through 5 in horizontal and vertical formats (A)
*state multiplication facts by filling in the blanks in a multiplication table (A)
*use the commutative property of multiplication (A)

3.04.02 The identity element (1) for multiplication and the role of zero

The student will be able to...

*recognize one (1) as the identity element in multiplication (A)
*find the product when zero (0) is a factor (A)

3.04.03 Multiplication of twos

The student will be able to...

*multiply two (2) by the numbers one (1) through nine (9) using vertical and horizontal formats (A)

*apply the commutative property of multiplication to the two times table (A)

*solve word problems related to skills learned in this module (A)

3.04.04 Multiplication of fives

The student will be able to...

*multiply five (5) by the numbers one (1) through nine (9) using vertical and horizontal formats (A)

*apply the commutative property of multiplication to the fives times table (A)

*solve word problems related to skills learned in this module (A)

3.04.05 Multiplication of threes and fours

The student will be able to...

*multiply three (3) by the numbers one (1) through nine (9), using vertical and horizontal formats (A)

*multiply four (4) by the numbers one (1) through nine (9), using the vertical and horizontal formats (A)

*apply the commutative property of multiplication to the 3 and 4 times tables (A)

*solve word problems related to skills learned in the module (A)

3.04.06 Multiplication of sixes and sevens

The student will be able to...

*multiply six (6) by the numbers one (1) through nine (9) using vertical and horizontal formats (A)

*multiply seven (7) by the numbers one (1) through nine (9) using vertical and horizontal formats (A)

*apply the commutative property of multiplication to the six (6) and seven (7) times table (A)

*solve word problems related to skills learned in this module (A)

3.04.07 Multiplication of eights and nines

The student will be able to...

* multiply eight (8) by the numbers one (1) through nine (9) using vertical and horizontal formats (A)
* multiply nine (9) by the numbers one (1) through nine (9) using vertical and horizontal formats (A)
*apply the commutative property of multiplication to the eight and nine times tables (A)
*solve word problems related to skills learned in this module (A)

3.04.08 Multiplication of tens

The student will be able to...

*multiply ten (10) by the numbers one (1) through nine (9) using vertical and horizontal formats (A)
*apply the commutative property of multiplication to the ten times table (A)
*apply the associative property of multiplication to the ten times table (A)

3.04.09 Patterns in multiplication tables through 10 x 10

The student will be able to...

*fill in a multiplication table from zero through 10 zeros to ten through 10 tens (A)
*find patterns in multiplication tables from (A)

3.04.10 Multiplication of 2- and 3-digit numbers by a 1-digit multiplier without regrouping (exchange)

The student will be able to...

* multiply multiplies of 100 by a 1-digit number without regrouping (exchange) (B)
* multiply 2- and 3-digit numbers by numbers less than 10 in vertical and horizontal formats without regrouping (exchange) (B)
*solve word problems related to skills learned in this module (B)

3.04.11 Multiplication through 3-digit numbers by a 1-digit multiplier with regrouping (echange)

The student will be able to...

*multiply a 2- or 3-digit number by a 1- digit number with regrouping (exchange) using the partial product method (B)

*use the short multiplication algorithm to multiply a 2- or 3-digit number by a 1-digit number with regrouping (exchange) (B)

*solve word problems related to skills learned in this module (B)

4.04.01 Basic multiplication facts for 11 and 12 through 144

The student will be able to...

*recognize the basic multiplication facts for 11 and 12 through 144 (A)

*solve word problems related to skills learned in this module (A)

4.04.02 Multiplication of 2- and 3-digit numbers by a 1-digit number

The student will be able to...

*multiply 10 and 100, by a 1-digit number (A)

*multiply multiples of 10 and 100 by a 1-digit number (A)

*use the distributive property of multiplication over addition (A)

*multiply 2- and 3-digit numbers by a 1-digit number without regrouping (A)

4.04.03 Multiplication of money values

The student will be able to...

*round values to the nearest dollar to estimate products (A)

*multiply values to $.99 by a 1-digit multiplier (A)

*multiply values to $9.99 by a 1-digit multiplier (A)

*solve word problems related to skills learned in this module (A)

4.04.04 Multiplication of 4-and 5-digit numbers by a 1-digit number

The student will be able to...

- *multiply 1,000 and 10,000 and multiples of 1,000 and 10,000 by a 1-digit number (A)
- *multiply 4-digit numbers by a 1-digit number without regrouping (exchange) (A)
- *multiply 4-digit numbers by a 1-digit number with regrouping (exchange) (A)
- *solve word problems related to skills learned in this module (A)

4.04.05 Multiplication of 2-digit numbers

The student will be able to...

- *multiply 10, and multiples of 10, by 10 and multiples of 10 (A)
- *multiply any 2-digit number by 10 and multiples of 10 (A)
- *multiply a 2-digit number by a 2-digit number without regrouping (A)
- *multiply a 2-digit number by a 2-digit number with regrouping (A)
- *solve word problems related to skills learned in this module (A)

4.04.06 Multiplication of a 3-digit number by a 2-digit number

The student will be able to...

- *multiply 100 and multiples of 100 by 10 and multiples of 10 (A)
- *multiply any 3-digit number by 10 and multiples of 10 (A)
- *multiply a 3-digit number by a 2-digit number (A)
- *solve word problems related to skills learned in this module (A)

5.04.01 Multiplication of multiples of ten by multiples of ten

The student will be able to...

*multiply multiples of ten in horizontal format (B)
*multiply multiples of ten in vertical format (A)
*solve word problems related to skills learned in this module (A)

5.04.02 Multiplication of two 2-digit numbers

The student will be able to...

*use the distributive property of multiplication over addition to multiply two 2-digit numbers in horizontal format (B)
*multiply a 2-digit number by a 2-digit number in vertical format (A)

5.04.03 Multiplication of a 3-digit number by a 2-digit number

The student will be able to...

*multiply multiples of ten by multiples of 100 in vertical format (B)
*use the distributive property of multiplication to multiply a 3-digit number by a 2-digit number in horizontal format (B)
*multiply a 3-digit number by a 2-digit number in vertical format (A)
*solve word problems related to skills learned in this module (A)

5.04.04 Multiplication of a 4-digit number by 2-digit numbers

The student will be able to...

*multiply multiples of 1,000 by multiples of ten (A)
*use the distributive property of multiplication to multiply a 4-digit number by a 2-digit number in horizontal format (B)
*multiply a 4-digit number by a 2-digit number in vertical format (A)
*solve word problems related to skills learned in this module (A)

5.04.05 Multiplication of 3-digit and 4-digit numbers by 3-digit numbers

The student will be able to...

*multiply 3- and 4-digit numbers by a 3-digit number (B)
*solve word problems related to skills learned in this module (B)

6.04.01 Multiplication by 4- or more digit numbers

The student will be able to...

*multiply a 4- or more digit number by a 4- or more digit number in vertical format (A)

*solve word problems related to skills learned in this module (A)

7.04 01. Multiplication of large numbers

The student will be able to...

*compute the product of a 1-digit , 2-digit and a 3-digit multiplier (A)

*compute the product of a whole number multiplied by powers of ten (A)

*apply the properties of multiplication: commutativity, associativity, distributivity (multiplication over addition), multiplicative property of zero, and the identity element (A)

8.04.01 Multiplication of powers with a common base

The student will be able to...

*multiply powers of ten by adding the exponents (B)

*multiply powers with a common base (B)

DIVISION OF WHOLE NUMBERS

Topic 05

2.05.01 Introduction to division

The student will be able to...

- *demonstrate that division is a process of sharing (B)
- *demonstrate that division is the process of repeated subtraction (B)
- *solve story problems related to skills learned in this module (B)

2.05.02 Division facts for twos through 10

The student will be able to...

- *seperate up to 10 objects into groups of two without a remainder (A)
- *determine the number of groups of two that can be formed from an even numbered group of objects up to 10 (A)
- *solve story problems related to skills learned in this module (A)

2.05.03 Division facts for fives through 25

The student will be able to...

- *separate up to 25 objects into groups of five without a remainder (A)
- *identify the number for a given group through 25 and tell how many groups of five are in that larger group (A)
- *solve story problems related to skills learned in this module (A)

2.05.04 Division facts for threes through 15

The student will be able to...

- *separate up to 15 objects into groups of three without a remainder (A)
- *identify the number for a given group (through 15) and tell how many groups of three are in that group (B)
- *solve story problems related to skills learned in this module (B)

2.05.05 Division facts for fours through 20

The student will be able to...

*separate up to 20 objects into groups of four, without a remainder (A)

*identify the number for a given set through 20 and tell how many groups of four are in that set (B)

*solve story problems related to skills learned in this module (B)

2.05.06 Division and multiplication as inverse operations

The student will be able to...

*recognize the division symbol ( ) (A)

*recognize that multiplication and division are inverse (opposite) operations (A)

*demonstrate related facts for number families (A)

3.05.01 Concepts of division

The student will be able to...

*demonstrate various ways to write a division example (A)

*name the dividend, the divisor and the quotient (A)

3.05.02 Division of whole numbers for twos through 18, for fives through 45

The student will be able to...

*divide by two for dividends through 18 without a remainder (A)

*divide by five for dividends through 45 without a remainder (A)

*recite the division facts for twos and fives (A)

*solve word problems related to skills learned in this module (A)

3.05.03 Division of whole numbers for threes through 27, for fours through 36

The student will be able to...

*divide by 3 for dividends up to 27 without a remainder (A)

*divide by 4 for dividends up to 36 without a remainder (A)

*solve word problems related to skills learned in this module (A)

3.05.04 Division of whole numbers for sixes through 54, for sevens through 63

The student will be able to...

*divide by 6 for dividends to 54 without a remainder (A)
*divide by 7 for dividends to 63 without a remainder (A)
*solve word problems related to skills learned in this module (A)

3.05.05 Division of whole numbers for eights through 72, for nines through 81

The student will be able to...

*divide by 8 for dividends to 72 without a remainder (A)
*divide by 9 for dividends to 81 without a remainder (A)
*solve word problems related to skills learned in this module (A)

3.05.06 Division of 2- and 3-digit dividends by 1-digit divisors without regrouping (exchange)

The student will be able to...

*estimate the quotient in a division example as a multiple of 10 (B)
*divide a 2-digit dividend by a 1-digit divisor without regrouping (exchange) or a remainder (B)
*divide a 3-digit dividend by a 1-digit divisor without regrouping (exchange) or a remainder (B)
*solve word problems related to skills learned in this module (B)

3.05.07 Division with remainders

The student will be able to...

*recognize a remainder as that which is "left over" in division (B)
*solve word problems related to skills learned in this module (B)

4.05.01 Properties of division

The student will be able to...

*use one (1) as a divisor (A)
*recognize that using zero as the divisor is meaningless, because the quotient will always be undefined (A)
*recognize that a dividend of zero divided by any non-zero number will always have a quotient of zero (A)

4.05.02 Division of a 2- and 3-digit dividend by a 1-digit divisor without regrouping

The student will be able to...

- *recognize a remainder as that which is "left over" in division (A)
- *estimate the quotients for division examples (A)
- *divide a 2- or 3-digit dividend by 1-digit divisor without regrouping (exchange) or a remainder and check (A)
- *divide a 2- or 3-digit dividend by a 1-digit divisor without regrouping (exchange) with a remainder and check (A)
- *solve word problems related to skills learned in this module (A)

4.05.03 Division of a 2- and 3-digit dividend by a 1-digit divisor with regrouping

The student will be able to...

- *divide a 2- or 3-digit dividend by a 1-digit divisor regrouping tens as ones with or without a remainder and check (A)
- *divide a 3-digit dividend by a 1-digit divisor regrouping hundreds as tens and tens as ones with or without a remainder and check (A)
- *use zero as placeholder in a quotient (A)
- *solve word problems using skills learned in this module (A)

4.05.04 The distributive property of division

The student will be able to...

- *identify the distributive property of division over addition or subtraction (B)
- *divide a 2- or 3-digit number by a 1-digit divisor without regrouping using the distributive property of division over addition (B)

4.05.05 Division of money values

The student will be able to...

- *round to the nearest dollar to estimate quotients (A)
- *divide values to $.99 by a 1-digit divisor and check (A)
- *divide values to $9.99 by a 1-digit divisor and check (A)
- *solve word problems related to skills learned in this module (A)

4.05.06 Division of 4-digit dividends by a 1-digit divisors

The student will be able to...

*divide a number through 9,999 by a 1-digit divisor and check using multiplication (B)

*solve word problems related to skills learned in this module (B)

4.05.07 Division of 2- and 3-digit dividends by multiples of 10 as divisors

The student will be able to...

*divide 2-digit dividends by 10 or a multiple of 10 and check (A)

*divide 3-digit dividends by 10 or multiples of 10 and check (A)

*solve word problems related to skills learned in this module (A)

4.05.08 Division of 2- and 3-digit dividends by 2-digit divisors

The student will be able to...

*divide 2-digit dividends by a 2-digit divisor and check (A)

*divide 3-digit dividends by a 2-digit divisor and check (A)

*solve word problems related to skills learned in this module (A)

5.05.01 Division of 5- and 6-digit dividends by 1-digit divisors

The student will be able to...

*divide 5- and 6-digit numbers using the distributive property of division over addition by a 1-digit divisor and check (B)

*divide 5- and 6-digit numbers using repeated subtraction by a 1-digit divisor and check (A)

*solve word problems related to skills learned in this module (A)

5.05.02 Division of 2-digit divisors with 5-and 6-digit dividends by 2-digit divisors

The student will be able to...

*estimate quotients(B)

*divide 5- and 6-digit numbers by a 2- digit number and check (A)

*solve word problems related to skills learned in this module (A)

5.05.03 Division by multiples of 100

The student will be able to...

*divide a dividend that is a multiple of 1,000 or 10,000 by a divisor that is a multiple of 100 (B)

*divide any 4- and 5-digit number using a divisor that is a multiple of 100 and check(B)

*solve word problems related to skills learned in this module (B)

5.05.04 Division by 3-digit divisors

The student will be able to...

*estimate the quotients (B)

*divide a 4-, 5- and 6-digit number by a 3-digit divisor and check (B)

*solve word problems related to skills learned in this module (B)

6.05.01 Division with large numbers

The student will be able to...

*divide a dividend with a zero in any place and check (A)

*divide a dividend that is at least a 4-digit number by a 4-digit divisor and check (A)

*solve word problems related to skills learned in this module (A)

7.05.01 Express a remainder as a fraction

The student will be able to...

*express a remainder in a division example as a fraction (A)

*solve verbal problems related to skills learned in this module (A)

8.05.01 Division of powers with a common base

The student will be able to...

*divide powers of ten by subtracting the exponents (B)

*divide powers with a common base (B)

FRACTIONS

Topic 06

0.06.01 **A whole unit**

The student will be able to...

- *identify an object as a unit when so considered
- *identify a part of a whole
- *distinguish part of a unit from the whole unit

0.06.02 **Share objects**

The student will be able to...

- *recognize that when a group is separated into two parts of the same size, each part is 1/2 of the whole group
- *solve oral story problems related to skills learned in this module

0.06.03 **Compare a half unit to a whole unit**

The student will be able to...

- *recognize that a whole object is greater than one-half (1/2) of the object
- *recognize that one-half (1/2) of an object is less than a whole

0.06.04 **The fraction one-half (1/2)**

The student will be able to...

- *recognize equal parts of a whole
- *recognize that when an object is separated into two (2) pieces of the same size, each piece is one-half (1/2) of the whole object
- *divide representative objects into halves
- *solve oral story problems related to skills learned in this module

1.06.01 Halves

The student will be able to...

*separate representative objects into halves (B)

*identify, write and record the symbol for one-half as 1/2, for two-halves as 2/2 (B)

*solve story problems related to skills learned in this module (B)

1.06.02 Fourths

The student will be able to...

*separate representative objects into fourths (B)

*identify and write fractional notation for one-fourth as 1/4, for two-fourths as 2/4, for three-fourths as 3/4, for four-fourths as 4/4 (B)

*identify 1/2 as equal to 2/4 pictorially, and record using the equal symbol (=) (B)

*solve story problems related to skills learned in this module (B)

1.06.03 Thirds

The student will be able to...

*separate representative objects into thirds (B)

*identify, write and record fractional notation for one-third as 1/3, for two-thirds as 2/3, for three-thirds as 3/3 (B)

*solve story problems related to skills learned in this module (B)

2.06.01 Fractions - fifths, sixths, and tenths

The student will be able to...

*recognize that when an object is separated into parts of of equal size, each part may be written as a unit fraction of the whole object (A)

*separate a concrete object into fifths, sixths, and tenths and identify the parts as equal (A)

*identify and write fractional notation for fifths through 5/5, sixths through 6/6 and tenths through 10/10 (A)

*solve story problems related to skills learned in this module (A)

2.06.02 Identify fractional parts of a group (1/2,1/3, 1/4)

The student will be able to...

- *separate a group through 10 objects into two equal groups using fractional parts (A)
- *separate a group through 20 objects into four equal groups using fractional parts (A)
- *separate a group through 15 objects into three equal groups using fractional parts (A)
- *solve story problems related to skills learned in this module (A)

3.06.01 Equivalent fractions

The student will be able to...

- *identify and define the numerator and denominator in a given fraction (A)
- *identify and record the total number of parts to equal one whole (halves, thirds, fourths, sixths, and eighths) (A)

3.06.02 Non-unit fractional parts of a group

The student will be able to...

- *separate a number of objects into non-unit fractional parts (B)
- *solve word problems related to skills learned in this module (B)

3.06.03 Addition and subtraction of fractions with like denominators

The student will be able to...

- *add fractions having like denominators in vertical and horizontal formats (B)
- *subtract fractions having like denominators in horizontal and vertical formats (B)
- *solve story problems related to skills learned in this module (B)

3.06.04 Ratio

The student will be able to...

- *write a ratio to represent a relationship between two quantities (A)
- *solve word problems related to skills learned in this module (B)

3.06.05 Improper fractions and mixed numbers

The student will be able to...

*define and identify an improper fraction (B)
*define and identify a mixed number (B)
*change an improper fraction to a mixed number (B)
*change a mixed number to an improper fraction (B)

4.06.01 Addition and subtraction of fractions with common denominators

The student will be able to...

*add fractions with common denominators (A)
*subtract fractions with common denominators (A)
*solve word problems related to skills learned in this module (A)

4.06.02 Multiplication of fractions

The student will be able to...

*multiply fractions having values less than 1 (B)
*rename any whole number as a fraction with a denominator of 1 (B)
*multiply a whole number by a fraction (B)

4.06.03 Equivalent fractions

The student will be able to...

*recognize equivalent fractions (A)
*match a set of equivalent fractions with its corresponding points on the number line (A)
*use the multiplication identity to generate equivalent fractions (B)

4.06.04 Order of unit fractions

The student will be able to...

*order numerically 3 unit fractions from least to greatest or greatest to least (A)
*solve word problems related to skills learned in this module (A)

4.06.05 Improper fractions and mixed numbers

The student will be able to...

- *identify and define improper fractions and mixed numbers (A)
- *change an improper fraction to a mixed number (B)
- *change a mixed number to an improper fraction (B)

5.06.01 Equivalent fractions

The student will be able to...

- *use the multiplicative identity of 1 for fractions to convert a given fraction to an equivalent fraction (A)
- *rename fractions as equivalent fractions (in simplest terms) by dividing the numerator and denominator by the same factor (A)
- *identify a number between any two fractional number (betweenness) (B)
- *rename fractions in simplest terms (A)

5.06.02 Greatest common factor (GCF) least common multiple (LCM)

The student will be able to...

- *determine the greatest common factor (GCF) of two or more numbers using the set method (A)
- *determine the least common multiples (LCM) of two or more numbers, using the set method (A)

5.06.03 Multiplication of fractions

The student will be able to...

- *multiply fractions having values less than 1 (A)
- *rename any whole number as a fraction with a denominator of 1 (A)
- *multiply a whole number by fraction and rename in simplest terms (A)
- *solve word problems related to skills learned in this module (A)

5.06.04 Multiplication of mixed numbers

The student will be able to...

- *multiply a whole number by a mixed number using horizontal format and rename in simplest terms (A)
- *multiply a mixed number by a mixed number and rename in simplest terms (B)
- *solve word problems related to skills learned in this module (A)

5.06.05 Addition of fractions with unlike denominators

The student will be able to...

- *find the least common denominator (A)
- *add mixed numbers with unlike denominators and rename in simplest form (A)
- *add several fractions and/or mixed numbers and rename in simplest form (A)
- *solve word problems related to skills learned in this module (A)

5.06.06 Subtraction of fractions with unlike denominators

The student will be able to...

- *subtract two fractions with like numerators and unlike denominators and rename in simplest form (A)
- *subtract mixed numbers and rename in simplest form (A)
- *solve word problems related to skills learned in this module (A)

5.06.07 Division of fractions

The student will be able to...

- *divide a whole number by a fraction using representative materials (A)
- *name the reciprocal of any fraction (B)
- *divide a whole number by a fraction renaming in simplest terms (multiplying by the reciprocal) (B)
- *solve word problems related to skills learned in this module (B)

5.06.08 Division of mixed numbers

The student will be able to...

- *divide a whole number by a mixed number and rename in simplest terms (B)
- *solve word problems related to skills learned in this module (B)

6.06.01 Factor primes

The student will be able to...

- *rename even numbers greater than 2 as the sum of two primes (B)
- *rename composite numbers as the product of primes (A)
- *use prime factorization to find the greatest common factor (A)

6.06.02 Estimation with fractions

The student will be able to...

- *estimate the result of addition and subtraction of fractions before computation (A)
- *solve word problems related to skills learned in this module (A)

6.06.03 Subtraction of fractions

The student will be able to...

- *subtract a mixed number from a whole number with simplification (A)
- *subtract fractions having unlike denominators with simplification (A)
- *check solutions using the principle of related operations (B)
- *solve word problems related to skills learned in this module (A)

6.06.04 Ratio

The student will be able to...

- *compare two numbers by naming the ratio of one to another and write them as a fraction (A)
- *write equivalent ratios for a given ratio (A)

6.06.05 Properties of multiplication with fractions

The student will be able to...

- *recognize that the commutative property of multiplication can be applied to multiplication of fractions (A)
- *recognize that the distributive property of multiplication can be applied to multiplication of fractional numbers (A)
- *estimate and compute the result of multiplication of fractions and mixed numbers (B)
- *solve word problems related to skills learned in this module (A)

6.06.06 Division of fractions

The student will be able to...

- *estimate the result of division of fractions before computation (A)
- *divide a fraction by a fraction (A)

6.06.07 Division of mixed numbers

The student will be able to...

*divide a mixed number by a fraction (A)
*divide a mixed number by a mixed number (A)
*solve word problems related to skills learned in this module (A)

7.06.01 Advanced fraction computations and concepts

The student will be able to...

*find the average of two or more fractions (C)
*raise a fraction to an indicated power (C)

7.06.02 Division with fractions

The student will be able to...

*compute the quotient of two fractions using the algorithm (A)
*solve verbal problems related to skills learned in this module (A)

7.06.03 Ratio and proportion

The student will be able to...

*compare two numbers by naming the ratio of one to another (A)
*write equivalent ratios for a given ratio (A)
*identify a proportion as a statement of equality between two ratios (A)
*identify means and extremes in a proportion (A)
*determine if two ratios form a true proportion (A)
*solve verbal problems related to skills learned in this module (A)

7.06.04 Meaning of percent

The student will be able to...

*convert a fraction whose denominator is a factor of 100 to a decimal and a percent (A)
*convert a mixed number to a decimal and a percent (A)
*convert a whole number percent to decimal fraction (A)

7.06.05 Verbal problems involving percent

The student will be able to...

*solve a verbal problem to find the amount of discount (A)
*solve a verbal problem to find the amount of sales tax (A)

8.06.01 Properties of fractions

The student will be able to...

- *use the commutative and associative properties of addition with fractions (A)
- *use the commutative and associative properties of multiplication with fractions (A)
- *recognize the additive identity property and the additive identity element 0 (A)
- *recognize the multiplicative identity property and the multiplicative identity element 1 (A)
- *recognize the additive inverse property and the additive inverse of a fraction (A)
- *recognize the multiplicative inverse property and the multiplicative inverse element of a fraction (A)
- *use the distributive property of multiplication over addition with fractions (A)

8.06.02 Positive and negative fractions

The student will be able to...

- *find negative and positive fractions on a number line (A)
- *determine that -a/b is equivalent to -a/b and a/-b (A)
- *compare two fractions and/or two mixed numbers using >, < and = (B)
- *order a set of fractions (B)

8.06.03 Proportions

The student will be able to...

- *solve a verbal problem with proportions and price rates (A)
- *solve a verbal problem involving proportion and rate, time and distance (A)
- *solve verbal problems involving proportion and scale drawings (A)
- *solve a verbal problem involving proportions (A)

8.06.04 Percents

The student will be able to...

- *convert a mixed number or decimal to a percent greater than 100% (A)
- *convert a percent greater than 100% to a mixed number or decimal (A)
- *convert a decimal to a percent less than 1% (A)
- *convert a percent less than 1% to a decimal (A)
- *recognize common fraction/decimal equivalents (A)

8.06.05 Using percents to solve problems

The student will be able to...

- *use percents to solve verbal problems involving compound interest (A)
- *use percents to solve a problem found in a newspaper (A)

DECIMALS

Topic 07

3.07.01 Decimals

The student will be able to...

- *identify and read a decimal (B)
- *write tenths as decimals (B)
- *count by tenths (B)

3.07.02 Addition and subtraction of decimals to tenths

The student will be able to...

- *add decimals in horizontal and vertical formats without regrouping (exchange) (B)
- *subtract decimals in horizontal and vertical formats without regrouping (exchange) (B)
- *solve word problems related to skills learned in this module (B)

4.07.01 Decimals to hundredths

The student will be able to...

- *identify and read a decimal (A)
- *write tenths as decimals (A)
- *count by tenths (A)
- *write any fraction with a denominator of 100 as a decimal (A)
- *identify the place value of digits in decimals through hundredths (A)

4.07.02 Order decimals

The student will be able to...

- *compare and order two or more decimals using the relation symbols < (less than) , > (greater than), = (equal to) (A)
- *recognize equivalency of decimals in tenths and hundredths (A)

4.07.03 Addition and subtraction of decimals to hundredths

The student will be able to...

- *add decimals to hundredths (A)
- * subtract decimals to hundredths (A)
- *solve word problems related to skills learned in this module (A)

4.07.04 Multiplication and division of decimals to tenths

The student will be able to...

- *multiply decimals to tenths (A)
- *divide decimals with dividends to tenths and whole number divisors (A)
- *solve word problems related to skills learned in this module (A)

5.07.01 Decimals- thousandths

The student will be able to...

- *identify the place value of decimals through thousandths (A)
- *compare decimals expressed as tenths, hundreths, and thousandths, using the symbols >, <, = (B)
- *round decimals expressed as hundredths to the nearest whole number (A)

5.07.02 Addition and subtraction of decimals through thousandths

The student will be able to...

- *add and subtract decimals through thousandths and check (A)
- *solve word problems related to skills learned in this module (A)

5.07.03 Multiplication of decimals through hundredths

The student will be able to...

- *multiply decimals by 10 and 100 (A)
- *multiply decimals (A)
- *solve word problems related to skills learned in this module (A)

5.07.04 Division of decimals through hundredths

The student will be able to...

- *divide a decimal by a whole number through hundredths (A)
- *use the identity element (1) to obtain equivalent decimal fractions (A)
- *solve word problems related to skills learned in this module (A)

6.07.01 Decimal notations

The student will be able to...

- *change fractions with denominators that are factors of of multiples of ten to equivalent decimals (1/2,1/5,1/4, 1/20,1/25,1/50, etc.) (A)
- *write any proper fraction or mixed numeral with a denominator of 1000 as a decimal fraction (A)
- *change fractions with denominators unrelated to multiples of ten, to equivalent decimals (1/3,1/8,1/6, etc.) (A)

6.07.02 Compare numbers in decimal notation

The student will be able to...

- *compare a given decimal by hundredths or thousandths (A)
- *solve word problems related to skills learned in this module (A)

6.07.03 Round decimals

The student will be able to...

- *round decimals in hundredths to the nearest tenth (A)
- *round decimals in thousandths to the nearest hundredth (A)

6.07.04 Percent

The student will be able to...

- *write a percent as a decimal (A)
- *express fractions and decimals as an equivalent percent(A)
- *solve word problems related to skills learned in this module (A)

6.07.05 Add numbers in decimal form

The student will be able to...

- *add decimals through ten thousandths and check (A)
- *solve word problems related to skills learned in this module (A)

6.07.06 Subtract numbers in decimal form

The student will be able to...

- *subtract decimals through ten thousandths and check (A)
- *solve word problems related to skills learned in this module (A)

6.07.07 Multiplication of decimals

The student will be able to...

- *multiply a decimal by 10, 100 or 1000 (A)
- *multiply a whole number by a decimal (A)
- *multiply a decimal by a decimal (A)
- *solve word problems related to skills learned in this module (A)

6.07.08 Division of decimals

The student will be able to...

- *divide decimals by a whole number (A)
- *divide decimals by a decimal fraction (A)
- *solve word problems related to skills learned in this module (A)

7.07.01 Decimal relationships

The student will be able to...

- *extend the base ten place value chart to include decimals whose denominators are powers of 10, and read and write decimal fractions in numerical and word form (A)
- *identify equivalent decimals (A)
- *compare and order decimals (A)
- *round off a given decimal to a specified place (A)
- *solve verbal problems related to skills learned in this module (A)

7.07.02 Addition of decimals

The student will be able to...

- *compute the sum of two or more decimals arranged horizontally (A)
- *solve verbal problems related to skills learned in this module (A)

7.07.03 Subtraction of decimals

The student will be able to...

- *compute the differences of two decimals arranged vertically with the minuend having fewer decimal places than the subtrahend (A)
- *solve verbal problems related to skills learned in this module (A)

7.07.04 Multiplication of decimals

The student will be able to...

- *determine and place the decimal point in the product of a given multiplication example if the digits of the product are given (A)
- *solve verbal problems related to skills learned in this module (A)

7.07.05 Division of decimals

The student will be able to...

- *compute the quotient by correctly placing the decimal point given a division example in which the divisor is a power of 10 (A)
- *compute the quotient to a specified decimal place given a division example in which the divisor is a whole number or a decimal (A)
- *solve verbal problems related to skills learned in this module (A)

7.07.06 Convert to decimals or fractions

The student will be able to...

- *convert a fraction to a terminating or non-terminating (repeating) decimal fraction (A)
- *convert a terminating or non-terminating (repeating) decimal to a fraction (A)

8.07.01 Scientific notation

The student will be able to...

- *write a standard base ten numeral in scientific notation (B)
- *write a number expressed in scientific notation as a standard base ten numeral (B)
- *multiply two numbers expressed in scientific notation (B)
- *divide two numbers expressed in scientific notation (B)

8.07.02 Rational numbers/terminating decimals

The student will be able to...

- *redefine a terminating decimal as a repeating decimal with a "0 repetend " (A)
- *define every rational number as a repeating decimal (A)

8.07.03 Irrational Numbers

The student will be able to...

- *define an irrational number as a non-repeating, non-terminating decimal (A)
- *distinguish between a rational and irrational numbers (A)
- *approximate to the nearest tenth the square root of non-perfect squares (A)
- *solve verbal/numerical problems related to skills learned in this module (A)

INTEGERS

Topic 08

4.08.01 Integers

The student will be able to...

- *identify a positive number as a number which appears to the right of zero on a number line (B)
- *identify a negative number as a number which appears to the left of zero on a number line (B)
- *identify situations when negative and positive numbers may be used (A)
- *solve word problems related to skills learned in this module (A)

5.08.01 Positive and negative numbers

The student will be able to...

- *use positive and negative numbers in real situations and games (B)
- *use the number line to solve problems relating to negative and positive numbers and identify zero as a neutral number (B)

6.08.01 Compare integers

The student will be able to...

- *identify +n as a positive number, -n as a negative number and (+n,-n) as a pair of opposites, assuming n > 0 (A)
- *identify which is the larger or smaller integer given any two intergers (A)
- *order a group of intergers in increasing or decreasing order (A)
- *solve word problems related to skills learned in this module (A)

6.08.02 Addition of integers

The student will be able to...

- *recognize that the sum of each negative integer and its opposite is 0 (B)
- *compute the sum of two positive integers (B)
- *compute the sum of two negative integers (B)
- *compute the sum of one negative integer and one positive integer (B)
- *solve word problems related to skills learned in this module (B)

7.08.01 Subtraction of signed numbers

The student will be able to...

- *subtract positive and negative integers using the inverse operation of addition (A)
- *subtract a negative integer from a negative integer using the inverse operation of addition (A)

7.08.02 Multiplication of signed whole numbers

The student will be able to...

- *multiply a positive whole number by another positive whole number (A)
- *multiply a negative whole number by a positive whole number (A)
- *multiply a negative whole number by another negative whole number (A)

7.08.03 Division of signed whole numbers

The student will be able to...

- *compute the quotient with a positive dividend and a positive divisor (B)
- *compute the quotient with a positive dividend and a negative divisor (B)
- *compute the quotient with a negative dividend and a positive divisor (B)
- *compute the quotient with a negative dividend and a negative divisor (B)

8.08.01 Properties with integers

The student will be able to...

- *use the commutative and associative properties of addition with integers (A)
- *use the commutative and associative properties of multiplication with integers (A)
- *recognize the additive identity property and the additive identity element 0 (A)
- *recognize the multiplicative identity element1(A)
- *recognize the additive inverse property and the additive inverse of an integer (A)
- *recognize the multiplicative inverse property and the multiplicative inverse of an integer (A)
- *use the distributive property of multiplication over addition with integers (A)

8.08.02 Division of signed numbers

The student will be able to...

- *compute the quotient with a positive dividend and a positive divisor (A)
- *compute the quotient with a positive dividend and a negative divisor (A)
- *compute the quotient with a negative dividend and a positive divisor (A)
- *compute the quotient with a negative dividend and a negative divisor (A)
- *raise an integer to a given power (A)

8.08.03 Roots/absolute value

The student will be able to...

- *find two square roots of a positive number (A)
- *find the cube root of a number (B)
- *define the absolute value of a number as the distance of its location on the number line from the origin (A)
- *find the absolute value of a number (A)

Geometry

Topic 09

0.09.01 Identification - parts of three-dimensional shapes

The student will be able to...

- *identify three-dimensional shapes
- *recognize faces, edges and corners of a three-dimensional shape

0.09.02 Shapes around us

The student will be able to...

- *recognize and form a circle
- *recognize and form a square
- *recognize and form a rectangle
- *recognize and form a triangle
- *identify and form patterns

1.09.01 Patterns

The student will be able to...

- *use attributes to classify objects by size, shape, or color (A)
- *identify different patterns with one variable attribute by size, shape, or color (A)

1.09.02 Properties of figures and shapes

The student will be able to...

- *observe the likenesses and differences of shapes in the environment (A)
- *discover some properties of 2-dimensional shapes (circles, squares, rectangles, and triangles) (A)
- *discover some properties of 3-dimensional shapes (cubes, rectangular prisms, spheres, cylinders and pyramids) (A)

1.09.03 Line symmetry

The student will be able to...

- *recognize and create shapes that have a line of symmetry (B)
- *discover whether or not a given line is a line of symmetry for a figure (halves) (C)

2.09.01 Points, lines, line segments, and rays around us

The student will be able to...

- *recognize points around us (B)
- *recognize lines around us (B)
- *recognize line segments around us (B)
- *recognize rays around us (B)

2.09.02 Explore properties of a circle, square, rectangle, and triangle

The student will be able to...

- *recognize and list some properties of a circle (round, continuous side, no corners, a center) (A)
- *recognize and list some properties of a square (four equal sides, four square corners) (B)
- *recognize and list some properties of a rectangle (four sides, four square corners) (B)
- *recognize and list some properties of a triangle (three sides, three corners) (B)

2.09.03 Symmetry

The student will be able to...

- *determine whether objects have lines of symmetry (A)
- *find more than one line of symmetry (A)

2.09.04 Attributes of three-dimensional objects

The student will be able to...

- *compare attributes such as size, shape, and weight of common three-dimensional objects (B)
- *compare attributes such as texture, or properties such as sliding or rolling (C)
- *recognize shapes that fit without gaps (C)

3.09.01 Points and lines

The student will be able to...

- *recognize and name a point (A)
- *identify and name a line (A)
- *identify and name a line segment (A)
- *identify and name a ray (A)

3.09.02 Construction of plane figures

The student will be able to...

- *use a compass and/or a protractor to construct circles (B)
- *use a ruler to construct rectangles and squares (B)
- *use a ruler to construct triangles (B)

3.09.03 Open and closed figures and curves

The student will be able to...

*recognize open and closed curves (B)

*recognize simple closed curves and complex closed curves (B)

3.09.04 Solid figures

The student will be able to...

*define and identify a sphere, cube, cone and cylinder (A)

*identify the vertex, edge, line segment, angle, and face of a solid figure (A)

3.09.05 Coordinate positions

The student will be able to...

*give direction for getting from one place to another on a grid using horizontal and vertical paths (B)

*identify points on a grid by using ordered pairs (B)

3.09.06 Polygons

The student will be able to...

*recognize and define a polygon (B)

*define quadrilateral and recognize the following as quadrilaterals: parallelogram, rectangle, square, rhombus trapezoid (B)

*define perimeter of a quadrilateral as the sum of the lengths of the sides of a quadrilateral (A)

4.09.01 Solid geometric figures (space figures) and their properties

The student will be able to...

*recognize and name various solid geometric figures (space figures) (A)

*identify solid geometric figures in the environment (A)

*identify and define faces, edges, angles, right angles, line segments, and vertices of space figures (A)

4.09.02 Lines and properties of lines

The student will be able to...

*identify intersecting lines as two lines having one and only one point in common (B)

*identify perpendicular lines as lines that form right angles (B)

*identify parallel lines as two lines that never intersect (B)

4.09.03 Properties of a circle

The student will be able to...

- *identify a radius of a circle (B)
- *identify the chords of a circle (B)
- *identify a diameter of a circle and compare the diameter of a circle to other parts of a circle (B)

4.09.04 Polygons

The student will be able to...

- *recognize and define a polygon (A)
- *recognize polygons in the environment (B)
- *recognize and define a diagonal of a polygon as a line segment joining any two nonconsecutive vertices (B)

4.09.05 Parallelograms

The student will be able to...

- *define a parallelodram as a quadrilateral in which the opposite sides are parallel (B)
- *recognize parallelograms in the environment (B)
- *define a square and a rectangle as parallelograms (B)

4.09.06 Symmetry

The student will be able to...

- *recognize lines of symmetry in geometric shapes (A)
- *recognize lines of symmetry in objects other than geometric shapes (A)

4.09.07 Coordinate geometry

The student will be able to...

- *locate positive and negative coordinates on a grid (A)
- *use line segments to connect points (A)

4.09.08 Geometric designs

The student will be able to...

- *construct geometric figures using straight-edge rulers, protractors and compasses (B)
- *use geometric figures to make designs and pictures (B)

5.09.01 Planes

The student will be able to...

*identify a plane as a flat surface which extends endlessly in all directions; it has the property that the line joining any two points of the plane lies entirely (B)

*identify intersecting planes (B)

*identify the intersections of planes and space figures (B)

5.09.02 Angles

The student will be able to...

*identify an angle (B)

*name angles using the angle symbol (B)

5.09.03 Angle measurement

The student will be able to...

*measure and/or construct right angles using a protractor (B)

*identify acute and obtuse angles (B)

*measure and/or construct acute and obtuse angles using a protractor (B)

5.09.04 Geometry - circles

The student will be able to...

*identify the central angle of a circle as an angle with its vertex at the center of the circle (B)

*identify the arc of a circle (B)

*construct a circle using a compass (B)

*identify an ellipse (B)

5.09.05 Congruent and similar figures

The student will be able to...

*recognize and construct congruent segments, angles and figures using a compass, protractor or ruler (B)

*recognize and construct similar figures (B)

5.09.06 Geometry - spheres

The student will be able to...

*recognize characteristics of a sphere (C)

*recognize that one-half of a sphere is called a hemisphere (C)

5.09.07 Geometric constructions

The student will be able to...

- *construct polygons through pentagons using rules and/or protractors (B)
- *construct a rhombus, a parallelogram and a trapezoid using a ruler and/or protractor (B)

6.09.01 Circles

The student will be able to...

- *identify and define terms relating to a circle (B)
- *describe and measure central angles (B)
- *construct angles given specific angular size in degrees using the protractor (A)

6.09.02 Triangles and angles

The student will be able to...

- *classify triangles according to the length of their sides (A)
- *find the sum of the angles of a triangle (A)
- *classify triangles according to their angles (A)
- *construct a triangle when given three line segments using a compass and a straight edge (A)

6.09.03 Space geometry - planes

The student will be able to...

- *identify the three possible positions for two lines in space (B)
- *identify the three possible positions for a line and a plane in space (B)

6.09.04 Solid geometric shapes

The student will be able to...

- *compare plane figures with solid figures (A)
- *recognize the differences between solid figures (A)
- *recognize that any face of a rectangular or triangular prism may be called the base (A)
- *compare triangular and rectangular prisms (A)

7.09.01 Angle classification

The student will be able to...

- *name and define the different types of angles (A)
- *define and identify adjacent, supplementary and complementary angles (A)

7.09.02 Vertical angles

The student will be able to...

- *use the results of measurements to identify vertical and supplementary angles (A)
- *use the generalization that when two lines intersect, vertical angles are equal (A)

7.09.03 Triangles

The student will be able to...

- *solve numerical problems involving the angles of a triangle (A)
- *determine that the sum of the length of any two sides of a triangle is greater than the length of the third side (A)
- *solve verbal problems related to skills learned in this module (A)

7.09.04 Circles

The student will be able to...

- *identify a diameter as the longest chord passing through the center of a circle (A)
- *use the relationship between the radius and the diameter ($D = 2R$) (A)
- *identify the circumference as the distance around the circle (A)
- *state the definitions of π as the ratio of the circumference of a circle to the diameter (A)
- *state the decimal and fractional approximation for π (A)
- *use the formula for circumference of a circle where the radius ($C = 2\pi r$) or the diameter ($C = \pi d$) is given (A)
- *solve verbal problems related to skills learned in this module (A)

7.09.05 Spatial figures

The student will be able to...

- *define a plane as being determined by (1) two intersecting or parallel lines, (2) three non-collinear points, or (3) a line and a point not on the line (B)
- *identify and count the number of faces, edges, vertices and bases in a given rectangular prism or triangular prism (A)

7.09.06 Parallel lines and their properties

The student will be able to...

- *identify a pair of parallel lines (A)
- *determine pairs of alternate interior angles and corresponding angles formed by a transversal through parallel lines (B)
- *apply the properties of parallel lines to numerical problems (B)

7.09.07 Angles of a polygon

The student will be able to...

- *find the sum of the angle measures of a polygon (B)
- *find the sum of the angles of any polygon in numerical problems (B)
- *identify a regular polygon (equilateral and equiangular) (B)
- *find each interior angle of a regular polygon in numerical problems (B)

8.09.01 Angles of a polygon

The student will be able to...

- *find the sum of the angles of a polygon (A)
- *find the sum of the angles of any polygon in numerical problems (A)
- *identify a regular polygon (equilateral and equiangular) (A)
- *find the measure of each interior angle of a regular polygon (A)

8.09.02 Parallel lines

The student will be able to...

- *determine pairs of alternate interior angles formed by a transversal crossing parallel lines (A)
- *determine pairs of corresponding angles formed by a transversal crossing parallel lines (A)
- *apply the properties of parallel lines to numerical problems (A)

8.09.03 Pythagorean relationship

The student will be able to...

- *know that the Pythagorean relationship is $a^2 + b^2 = c^2$ where a and b are legs of a right triangle and c is the hypotenuse (A)
- *determine if three given measurements can be the lengths of a right triangle (A)
- *determine the length of a side of a right triangle given the rational lengths of the other two sides (A)
- *solve verbal problems involving the Pythagorean relationship (A)

8.09.04 Congruence and similarity

The student will be able to...

- *define congruence and recognize its symbol (A)
- *name and find corresponding sides and angles of congruent triangles (A)
- *construct congruent segments and angles by using a compass, protractor or ruler (A)
- *recognize that two triangles are congruent by SSS $\cong$ SSS, SAS $\cong$ SAS, ASA $\cong$ ASA (A)
- *define similarity and recognize its symbol (A)
- *recognize that two triangles are similar if aaa $\cong$ aaa or aa $\cong$ aa (A)
- *recognize corresponding sides and corresponding angles of similar triangles (A)
- *recognize that corresponding sides of similar triangles are in proportion (A)
- *determine the length of a side of a triangle given a similar triangle (A)

8.09.05 Spatial figures

The student will be able to...

- *define a plane as being determined by (1) two intersecting or parallel lines, (2) three non-collinear points, or (3) a line and a point not on a line (A)
- *identify geometric solids (A)

MEASUREMENT

Topic 10

0.10.01 Non-standard measurement

The student will be able to...

- *measure the length and/or height of objects using nonstandard units of measure
- *measure the capacity of objects using nonstandard units of measure
- *define and compare nonstandard measurements of weight

0.10.02 Weight

The student will be able to...

- *distinguish between the concepts of heavy and light
- *compare objects that weigh more or less and the same as

0.10.03 Capacity of containers

The student will be able to...

- * identify containers that are full/empty, half full/half empty
- * distinguish among more than, less than, and the same as

0.10.04 Time of day

The student will be able to...

- *recognize times of the day - morning, noon, afternoon and night
- *distinguish between before/after, day/night, and early/late

0.10.05 Calendar

The student will be able to...

- *recognize that a calendar has the name of the month, days of the week, year and dates
- *recognize the time relationship - yesterday, today, and tomorrow
- *name the holidays and seasons as they occur

0.10.06 General temperature measurement terms

The student will be able to...

- *distinguish between the concepts of hot and cold
- *distinguish between cooler than and hotter than
- *distinguish between as cold as and as hot as

0.10.07 Size Concepts

The student will be able to...

- *distinguish among the degrees of size-large/larger/largest and small/smaller/smallest
- *distinguish among the concepts of length - long/longer/longest and short/shorter/shortest

0.10.08 Money: pennies

The student will be able to...

- *recognize and identify the value of the penny as 1 cent and the ¢ sign
- *count and write the value through 4 cents or 4¢
- *solve story problems related to skills learned in this module

0.10.09 Instruments of measurement

The student will be able to...

- *recognize that a ruler is an instrument used to measure length
- *recognize that a scale is an instrument used to measure weight (heavy/light)
- *recognize that a thermometer is an instrument used to measure temperature (hot/cold)

0.10.10 Time relationships

The student will be able to...

- *distinguish between it takes longer and less time than
- *recognize as long as
- *distinguish between older and younger
- *solve oral story problems related to skills learned in this module

0.10.11 Comparative size terms

The student will be able to...

- *distinguish between shorter than and as short as
- *distinguish between longer than and as long as
- *estimate which object is about as long as or as wide as
- *recognize the terms farther and nearer when referring to distance

0.10.12 Coins: nickel and dime

The student will be able to...

- *recognize and name a nickel
- *recognize and name a dime
- *discriminate among a penny, nickel and dime

0.10.13 Money: quarters through dollar bill

The student will be able to...

- *recognize and name a quarter
- *recognize and name a half dollar
- *recognize and name a dollar bill

1.10.01 Seasons of the year

The student will be able to...

- *name and sequence the seasons of the year (B)
- *name the holidays as they occur (B)
- *name the current season , the season that came before and the season that will soon follow (A)

1.10.02 Days of the week

The student will be able to...

- *name and sequence the days of the week (A)
- *identify what day of the week yesterday was, today is, and tomorrow will be (A)
- *name the current date (A)
- *solve story problems related to the skills learned in this module (B)

1.10.03 Measurement of length - non-standard

The student will be able to...

- *measure length using non-standard units (A)

1.10.04 Measurement of weight - non-standard

The student will be able to...

- *measure weight using non-standard units (A)

1.10.05 Size comparisons

The student will be able to...

- *distinguish and compare objects using the terms short/tall and short/long (A)
- *distinguish and compare objects using the terms long, longer, and longest; short, shorter, and shortest and tall, taller, and tallest (A)
- *distinguish and compare objects using the terms wide, wider, and widest and narrow, narrower, and narrowest (A)

1.10.06 Capacity

The student will be able to...

- *compare capacity of various size containers (B)
- *estimate the capacity of various size containers (B)

1.10.07 Pennies and nickels

The student will be able to...

- *identify and recognize that one nickel is equivalent to the value of five pennies (A)
- *solve story problems related to skills learned in this module (A)

1.10.08 Measure capacity

The student will be able to...

- *define and compare full, half full, and empty (A)
- *recognize a liter (B)

1.10.09 Temperature - Fahrenheit

The student will be able to...

- *read a thermometer using non-numerical expressions (A)
- *identify the degrees for the freezing point (32°F) and the boiling point (212°F) of water on a Fahrenheit thermometer (B)

1.10.10 Temperature - Centigrade

The student will be able to...

- *identify the degrees for the freezing point (0°C) and the boiling point (100°C) of water on a Celsius thermometer (B)

1.10.11 Time to the hour

The student will be able to...

- *identify the hour hand (short hand) and minute hand (long hand) of the clock (A)
- *read time on a standard and digital clock on the hour (o'clock) (A)

1.10.12 Standard measurement - customary

The student will be able to...

- *identify the inch, foot and yard as standard units of length (B)
- *use rulers and measuring tapes to measure familiar objects to the nearest inch, foot and yard (B)
- *identify ounce and pound as standard units of weight (B)
- *determine the weight of objects to the nearest ounce, pound (B)

1.10.13 Standard measurment - metric

The student will be able to...

- *identify the centimeter, decimeter and meter as standard units of length (B)
- *use centimeter rulers, centimeter measuring tapes and meter sticks to measure familiar objects (B)
- *identify the kilogram as a standard unit of weight (B)
- *determine the weight of objects to the nearest kilogram (B)

1.10.14 Time to the half-hour

The student will be able to...

- *identify time "past the hour" (B)
- *identify time "before the hour" (B)
- *read time on a standard and digital clock on the half-hour (B)

1.10.15 Dimes

The student will be able to...

- *identify equivalent values of a dime using nickels and pennies (A)
- *find the value of coins (pennies and nickels) having sums through 10¢ (A)
- *find the difference using minuends through 10¢ (A)
- *solve story problems related to skills learned in this module (A)

1.10.16 Quarters, half-dollars, dollars

The student will be able to...

- *recognize and identify a quarter, half dollar and dollar bill (B)
- *identify equivalent values of a quarter using dimes, nickels, and pennies (B)
- *read money through $9.99 using the dollar sign and decimal point (B)

2.10.01 Time - months and years

The student will be able to...

- *recite the months of the year in sequential order (A)
- *state what last month was and what next month will be (A)
- *identify the number of days in a month and in a year (B)
- *read the month, day, and year from a calendar (A)
- *identify the dates that are before and after a given date (B)

2.10.02 Quarters, half-dollars

The student will be able to...

- *recognize and identify a quarter and a half-dollar (A)
- *identify equivalent values of a quarter using dimes, nickels and pennies (A)
- *identify equivalent values of a half-dollar using quarters, dimes, nickels and pennies (B)
- *solve story problems related to skills learned in this module (A)

2.10.03 Money- dollars

The student will be able to...

- *identify the value of a dollar bill as 100¢ (A)
- *show equivalent values of a dollar bill (B)
- *write the decimal point and dollar sign to show one dollar symbolically (A)
- *transpose a given amount from the word form using the dollar sign, decimal point, and numbers (C)

2.10.04 Temperature - Celsius and Fahrenheit thermometers

The student will be able to...

- *explain the meaning of the word degree and identify its symbol (B)
- *recognize and read a Fahrenheit thermometer to the nearest five degrees, ten degrees (B)
- *identify the degree scale as shown on a Celsius thermometer and read to the nearest five degrees, ten degrees (B)

2.10.05 Linear measurement - inch and foot

The student will be able to...

- *identify various standard tools of linear measurement (A)
- *identify an inch on a ruler (A)
- *measure objects less than a foot in length to the nearest inch and record using the symbol for inch (") (B)
- *use the 12-inch ruler to measure a line segment (B)
- *identify the relationships between inch and foot (B)
- *measure objects to the nearest foot and record, using the symbol for feet (') (A)

2.10.06 Money - make change through the value of a dollar

The student will be able to...

- *make change through the value of fifty cents (B)
- *use combinations of coins to make change through the value of one dollar (B)

2.10.07 Linear measurement - meter, decimeter, and centimeter

The student will be able to...

- *measure objects using a meter stick and its subdivisions (cm/dm) (A)
- *recognize that 100 cm = 1 m (B)
- *compare centimeters, decimeters, and meters using equivalent measures (B)
- *solve story problems related to skills learned in this module (A)

2.10.08 Weight - ounce, pound

The student will be able to...

- *identify pounds and ounces as standard units of weight (A)
- *estimate and weigh familiar objects using appropriate customary units (A)

2.10.09 Weight - gram and kilogram

The student will be able to...

*identify grams and kilograms as metric units of weight (A)

*estimate and weigh familiar objects using appropriate metric units (A)

2.10.10 Linear measurement - half inch

The student will be able to...

*identify 1/2 inch on the 12 inch ruler (C)

*measure and record the lengths of objects to the nearest 1/2 inch (C)

*measure and record the lenghts of line segments to the nearest 1/2 inch (C)

2.10.11 Dozen and half dozen

The student will be able to...

*identify a group of twelve objects as a dozen and six objects as a half dozen (A)

*solve story problems related to skills learned in this module (A)

2.10.12 Time - quarter hour, minutes, and seconds

The student will be able to...

*identify relationships between seconds and minutes, and minutes and hours (B)

*read and write time to the quarter hour from a standard and a digital clock (B)

*solve story problems related to skills learned in this module (B)

2.10.13 Cup, pint, quart

The student will be able to...

*identify the relationships among cup, pint and quart (A)

*estimate and measure capacity using different customary units (A)

2.10.14 Deciliter

The student will be able to...

*identify relationships between a liter and a deciliter (C)

*estimate and measure capacity using different metric units (A)

3.10.01 Days and dates on a calendar

The student will be able to...

- *relate days and dates from a given calendar date (A)
- *identify number relationships among days, weeks, months and years (A)

3.10.02 Measurement - time

The student will be able to...

- *identify relationships between seconds and minutes, and minutes and hours (A)
- *read and write time in terms of quarter hours on standard and digital clocks (A)
- *tell time by five minute intervals (B)
- *tell time by minute intervals (before, to, of the hour) (B)
- *tell and record the time using A.M. and P.M. (A)
- *solve word problems related to skills learned in this module (B)

3.10.03 Temperature - Fahrenheit and Celsius Thermometers

The student will be able to...

- *recognize and read a Fahrenheit thermometer to the nearest five degrees, ten degrees (A)
- *recognize and read a Celsius thermometer to the nearest five degrees, ten degrees (A)

3.10.04 Weight in customary units

The student will be able to...

- *weigh familiar objects to the nearest whole pound, 1/2 pound and, 1/4 pound (B)
- *read various types of scales using customary units (A)

3.10.05 Weight in metric units

The student will be able to...

- *identify equivalent metric units of weight (B)
- *read various types of scales using metric units (A)

3.10.06 Inch, foot, yard

The student will be able to...

- *measure objects less than a foot in length to the nearest inch and record using the symbol for inch (") (A)
- *identify 1/2 inch on a ruler and measure objects to the nearest 1/2 inch (A)
- *identify 1/4 inch on a ruler and measure objects to the nearest 1/4 inch (A)
- *identify a foot as a unit of length and measure objects to the nearest foot (A)
- *identify a yard as a unit of length and measure objects to the nearest yard (A)
- *identify the numerical relationships among inch, foot, and yard (A)

3.10.07 Recognize and compare relationships (one dollar, five dollars and ten dollars)

The student will be able to...

- *show equivalent values of a dollar bill (A)
- *use combinations of coins to make change through the value of one dollar (A)
- *recognize and identify a five dollar bill and its equivalent values (A)
- *recognize and identify a ten dollar bill and its equivalent values (A)

3.10.08 Linear measurement - metric

The student will be able to...

- *identify a millimeter as a tenth of a centimeter and use the abbreviation mm (B)
- *identify equivalent metric units of length (B)

3.10.09 Capacity - metric

The student will be able to...

- *identify and compare the millimeter (ml) to the liter (l) (B)
- *identify equivalent metric units of capacity (B)

3.10.10 Measurement - speed

The student will be able to...

- *identify uniform speed as rate (C)
- *identify time as the number of hours traveled (C)
- *solve word problems related to skills learned in this module (C)

4.10.01 Money to fifty dollar bill

The student will be able to...

*recognize and identify ten, twenty, and fifty dollar bills, and their monetary values (A)

*solve word problems related to skills learned in this module (A)

4.10.02 Linear measurement

The student will be able to...

*identify 1/8 inch on the ruler (A)

*identify a mile as a customary measure of distance (A)

*identify the equivalent relationships among inch, foot, yard and mile (A)

*identify a kilometer as a metric measure of distance (A)

4.10.03 Customary capacity relationships

The student will be able to...

*identify and compare the number of ounces in a cup, a pint, and a quart (A)

*identify the relationships among cups, pints, quarts, and gallons (A)

*use measuring skills to follow the directions in a recipe (A)

4.10.04 Perimeters

The student will be able to...

*identify perimeter as the total distance around the sides of an object or space (A)

*find the perimeter of a rectangle (A)

*develop and use a formula for finding the perimeter of a rectangle (B)

4.10.05 Circumference

The student will be able to...

*define circumference as the distance around a circle (B)

*find the the relationship between diameter and circumference (B)

*find the relationship between radius and circumference (B)

4.10.06 Measurement - temperature

The student will be able to...

*read and locate the degrees for the freezing point of water (32 F), the boiling point of water (212 F), and normal body temperature (98.6 F) on a Fahrenheit thermometer (A)

*read and locate the degrees for the freezing point of water (0 C), the boiling point of water (100 C) and normal body temperature (37 C) on a Celsius thermometer (A)

4.10.07 Area

The student will be able to...

*define and measure the area of a rectangle by finding the number of square units contained in the rectangle (A)

*develop and use a formula to find the area of a rectangle (B)

4.10.08 Volume

The student will be able to...

*define and measure the volume of a rectangular prism (block) by finding the number of cubic units contained in the block (A)

*develop and use a formula to find the volume of a rectangular prism (block) (B)

5.10.01 Measurement - linear 1/16"

The student will be able to...

*identify 1/16 of an inch on a one foot ruler (B)

*measure to the nearest inch, half-inch, quarter-inch, eighth-inch, and sixteenth-inch (B)

5.10.02 Linear measurement - metric

The student will be able to...

*identify metric prefixes (A)

*solve word problems relating to skills learned in this module (A)

5.10.03 Perimeter

The student will be able to...

*compute the perimeter (A)

*solve word problems related to skills learned in this module (A)

5.10.04 Area of a rectangular region

The student will be able to...

*find the area (A) of a rectangle by multiplying length times the width (A)

*solve word problems relating to skills learned in this module (A)

5.10.05 Scale drawings

The student will be able to...

*identify the need for scale drawings (B)

*make a scale drawing of a room developing key (B)

5.10.06 Mass

The student will be able to...

*identify relationships among ounces (oz.), pounds (lbs.) and tons (t) (B)

*identify the relationships among gram (g), milligram (mg) and kilogram (kg) and the metric ton (A)

*solve word problems related to skills learned in this module (A)

5.10.07 Fractional linear measurement

The student will be able to...

*use fractional numbers to identify relationships among inches, foot, yard, mile (B)

*solve word problems related to skills learned in this module (B)

5.10.08 Time - fractional relationships

The student will be able to...

*identify the relationships among month and year (B)

*identify the relationships among seconds, minutes and hours (A)

*identify a decade (B)

*identify a century (B)

5.10.09 Distance

The student will be able to...

*recognize the formula for distance (C)

*solve word problems related to skills learned in this module (C)

6.10.01 Perimeter of polygons

The student will be able to...

- *find the perimeter of equilateral polygons (A)
- *find the perimeter of other polygons (A)
- *solve word problems related to skills learned in this module (A)

6.10.02 Operations with measures

The student will be able to...

- *convert hours and minutes to minutes (A)
- *multiply units of weight by a 1-digit multiplier and convert products to simplest form (A)
- *multiply units of liquid measure by a 1-digit multiplier and convert products to simplest form (A)
- *multiply units of linear measure by a 1-digit multiplier and convert products to simplest form (A)

6.10.03 Metrics

The student will be able to...

- *define meter, liter and gram using the following prefixes: deci (1/10 of a standard unit), centi (1/100 of a standard unit), milli (1/1000 of a standard unit), deka (10 standard units), hecto (100 standard units), kilo (1000 standard units) (A)
- *draw approximate comparisons between metric units and customary units of measure (A)

6.10.04 Circumference

The student will be able to...

- *construct a circle given the radius (B)
- *compute the diameter of a circle given its radius using the equation $d = 2r$ (A)
- *determine the relationship of the circumference of a circle to the diameter of a circle (A)

6.10.05 Area of polygons

The student will be able to...

- *measure the area of an object using standard or metric square units of measure (A)
- *determine and use the formula for finding the area of a rectangle (A)
- *find the area of parallelograms using the formula A = b x h (Area = base x height) (A)
- *find the area of any triangle given the base and the height using the formula 1/2 x b x h (Area = 1/2 x base x height) (A)

6.10.06 Volume

The student will be able to...

- *recognize the cubic inch as a standard unit of measure for space (A)
- *develop the formula and compute the volume of a rectangular prism (A)
- *determine the volume of a cylinder by using metric capacity measures (A)

6.10.07 Scale drawings

The student will be able to...

- *interpret the meaning of the legend shown on maps (A)
- *recognize illustrations drawn to scale (A)
- *make a scale drawing (A)

7.10.01 Measurement - linear (English)

The student will be able to...

- *select the appropriate unit for measuring length (A)
- *measure a given line segment with a standard ruler in English units to the nearest 1 in., 1/2 in., 1/4 in., 1/8 in. (A)
- *convert a given measure of length to another equivalent unit of length (A)
- *solve verbal problems related to skills learned in this module (A)

7.10.02 Computation with customary measurement

The student will be able to...

- *compute the sum of different units of length, time, weight, liquid measure (A)
- *compute the differences of a measurement given in linear units, time units, weight units, or liquid volume units (A)
- *compute the product of a measurement given in linear units, time units, weight units or liquid volume units (A)
- *compute the quotient of a measurement given in linear units, time units, weight units, or liquid volume units (A)
- *solve verbal problems related to skills learned in this module (A)

7.10.03 Metric units

The student will be able to...

- *select the appropriate unit of linear volume and weight measure (A)
- *describe the relationships between/among metric prefixes using base ten and identify the symbols for metric units (A)

7.10.04 Perimeter, area and volume

The student will be able to...

- *use the formulas for finding the perimeter of a rectangle, triangle, square (A)
- *find the area of polygons using the appropriate formulas (A)
- *compute the volume of a rectangular prism and a cube in standard units of measure using the formulas, V = L x W x H and V = B x H (where B = the area of the base) (A)
- *solve verbal problems related to skills learned in this module (A)

7.10.05 Perimeter of irregular figures

The student will be able to...

- *define and identify a semicircle
- *find the length of a semi-circumference (B)
- *find missing lengths of irregular figures by applying the properties involving opposite sides of a rectangle (B)
- *find the perimeter of irregular geometric figures (use π = 3.14) (B)

8.10.01 Perimeter

The student will be able to...

- *find the perimeter of a regular polygon (A)
- *solve verbal problems related to skills learned in this module (A)

8.10.02 Perimeter of irregular figures

The student will be able to...

- *define and identify a semicircle (A)
- *find the length of a semicircle (A)
- *find the missing lengths of irregular figures by applying the properties of a rectangle (A)
- *find the perimeter of irregular geometric figures (use ~ π 3.14) (A)

8.10.03 Area

The student will be able to...

- *compute the area of a right triangle (A)
- *compute the area of a parallelogram (A)
- *compute the area of a trapezoidal region using the formula $A = 1/2h\ (b_1 + b_2)$ (A)
- *compute the area of a circular region (use ~ π 3.14) (A)
- *solve verbal problems related to skills learned in this module (A)

8.10.04 Surface area

The student will be able to...

- *define and find the surface area of a rectangular prism or cube (B)
- *find the surface area of a cylinder (B)
- *find the surface area of a pyramid (B)
- *solve verbal problems related to skills learned in this module (B)

8.10.05 Volume

The student will be able to...

- *find the volume of a cube (A)
- *find the volume of a prism (A)
- *recognize the formula for and compute the volume of a pyramid (A)
- *find the volume of a right circular cylinder (A)
- *recognize the formula for and compute the volume and a cone (A)
- *recognize the formulas for and compute the volume of a sphere (B)
- *solve verbal problems related to skills learned in this module (A)

ALGEBRAIC CONCEPTS

Topic 11

1.11.01 Number Sentences

The student will be able to...

- *recognize a number sentence that is true or false (B)
- *read and complete number sentences with place holders (B)

2.11.01 Open Sentences

The student will be able to...

- *identify and solve an open sentence (A)
- *use tables to show number relationships (B)
- *use place holders to identify patterns of sums and differences (B)

3.11.01 Open Sentences

The student will be able to...

- *identify and solve open number sentences (A)
- *use tables to show number relationships (A)

3.11.02 Inequalities

The student will be able to...

- *read and write addition and multiplication inequalities (A)
- *solve inequalities involving addition and multiplication (A)

4.11.01 Open Sentences

The student will be able to...

- *identify an open sentence as a number sentence in which a letter or frame is used in place of a number (A)
- *solve an open sentence (A)
- *using the information from a given table, write the open sentence that could be used to find the missing information (B)

5.11.01 Order of Operations

The student will be able to...

- *identify and utilize the correct order of operations (A)
- *recognize that in the order of operations, any calculations indicated within parenthesis must be computed first (A)

6.11.01 Equations and Solution Sets

The student will be able to...

- *define and recognize an open sentences using variables and equal (=) symbols (A)
- *define equivalent open sentence as equations having the same solution set (A)

6.11.02 Graph Coordinates

The student will be able to...

- *complete a table of solutions for an equation with two variables (B)
- *graph solution sets of an equation with two variables (values greater than zero) (B)

7.11.01 Translating arithmetic word phases into numerical phrases

The student will be able to...

- *list the operations and/or symbols associated with words (A)
- *determine which operations have the commutative property (A)
- *translate a word phrase expressing a binary operation into a mathematical expression (A)

7.11.02 True and false numerical statements

The student will be able to...

- *state the order of operations as it applies to addition, subtraction, multiplication and division (A)
- *determine whether a numerical expression is a phrase or a sentence (A)
- *translate a word sentence into a numerical sentence including parentheses (B)
- *determine whether a numerical statement is true or false (A)

7.11.03 Variables

The student will be able to...

- *state the meaning of a variable (A)
- *determine the value of an expression (A)

7.11.04 Functions (relationships of variables

The student will be able to...

- *recognize a function in the form y = ax (B)
- *recognize a function in the form y = ax + b (B)

8.11.01 Algebraic expressions and sentences

The student will be able to...

*translate a verbal phrase into mathematical symbols (A)

*translate an open verbal sentence or a verbal statement into mathematical symbols (A)

* evaluate àn algebraic expression (A)

8.11.02 Solution sets

The student will be able to...

*identify sentences as either equations or inequalities (A)

*determine by inspection the solution set of an equation in one variable given a finite set of integers (A)

*determine by inspection the solution set of an equation in one variable given an infinite set of integers (A)

*determine the solution set of an equation in one variable given a finite set of rational numbers (A)

8.11.03 Inequalities

The student will be able to...

*determine the solution set of an inequality in one variable given a finite replacement set of integers (A)

*determine the solution set of an inequality in one variable given an infinite replacement set of integers (A)

*graph the solution set of an inequality on a number line (A)

8.11.04 First degree equations and inequalities

The student will be able to...

*use the reflexive, symmetric and transitive properties of equality (A)

*solve an equation in the form $x + b = c$ or $x-b=c$ (A)

*solve an equation in the form $ax = c$ or $x/a = c$ where $a = 0$ (A)

*solve an equation in the form $ax + b = c$, where $a = 0$ (A)

8.11.05 Graphs of first and second degree equations

The student will be able to...

*identify horizontal, vertical and oblique lines (B)

*make a table of values for equations in the form $y = ax$, $y = ax + b$ and $ax + by = c$ where $a = 0$ and $b = 0$ (B)

*graph a first degree equation in two variables from a table of values (B)

*make a table of values for equation in the form $y = ax2$ and $y = -ax2$ (B)

* graph a second degree equation in the form $y = ax2$ or $y = -ax2$ from a table of values (B)

* solve verbal problems related to skills learned in this module (B)

8.11.06 Coordinate planes - first quadrant

The student will be able to...

*graph an equation of the form $x = a$, $x = -a$, $y = a$, $y = -a$ (A)

*graph an inequality of the form $x > a$, $y > a$, $x < a$ and $y < a$ in the first quadrant (B)

*graph an inequality of the form $y > ax$, $y < ax$, $y > x + b$, $y < x + b$, $y > ax + b$ and $y < ax + b$ in the first quadrant (B)

*graph a compound inequality in the form $y \geq ax + b$ or $y \leq ax + b$ in the first quadrant (B)

*name an inequality of the form $y > a$, $y < a$, $y \geq a$, $y \leq a$, $x > a$, $x < a$, $x \geq a$, $x \leq a$ when given a graph in the first quadrant (B)

8.11.07 Coordinate planes - all quadrants

The student will be able to...

*graph an inequality of the form $x > a$, $y > a$, $x < a$ and $y < a$ in any quadrant (B)

*graph an inequality of the form $y > ax$, $y < ax$, $y > x + b$, $y < x + b$, $y > ax + b$ and $y < ax + b$ in any quadrant (B)

* graph a compound inequality in the form $y \geq ax + b$ or $y \leq ax + b$ in any quadrant (B)

*name an inequality in the form $x > a$, $x \geq a$, $y > a$, $y \geq a$, $x < a$, $x \leq a$, $y < a$ or $y \leq a$ when shown the graph (B)

GRAPHS, PROBABILITY, STATISTICS

Topic 12

0.12.01 Number relationships

The student will be able to...

- *construct a bar graph using blocks/cubes
- *transfer a graph to a pictorial representation
- *interpret pictorial representations as those representing concrete objects or people
- *identify number relationships using symbols and pictures
- *count aloud the number of squares that are colored on a bar graph
- *color the corresponding number of squares on a vertical bar graph when given concrete objects or the pictorial representation
- *write the numeral that represents the number of squares colored on a vertical and horizontal bar graph

0.12.02 Graph reading

The student will be able to...

- *construct and interpret a simple picture graph

0.12.03 Chance events/predicting

The student will be able to...

- *distinguish between certain and uncertain in regard to events
- *distinguish between more or less likely in chance situations
- *recognize the outcomes of activities as chance (guessing)

1.12.01 Pictographs

The student will be able to...

- *collect, organize and graph data relating to preference using two categories at a time (A)
- *read and interpret the information given in a graph using more than two categories (A)
- *use information on a graph to answer questions involving more or less (A)

1.12.02 Bar graphs

The student will be able to...

- *color the corresponding number of squares on vertical or horizontal bar graphs, when given concrete objects or the pictorial representation (B)
- *identify and record the number of squares that are colored on a bar graph (B)
- *gather, organize and record data based on personal or group experience to complete a bar graph using one-to-one correspondence (B)

1.12.03 Probability - chance events

The student will be able to...

- *predict the likelihood of events happening (B)
- *record the outcome of a chance event (B)

2.12.01 Data - collect, tabulate, arrange and compare in tables

The student will be able to...

- *collect, tabulate and arrange data in a table (A)
- *compare data in a table (A)

2.12.02 Bar graphs

The student will be able to...

- *collect and structure data based on personal or group experiences to complete a bar graph (A)
- *interpret a bar graph (A)

2.12.03 Probability - predict and tally outcomes

The student will be able to...

- *predict the ourcome of three equally likely events (B)
- *tally the outcome in a three-possibility event (B)

2.12.04 Arrangements of 2 through 5 items

The student will be able to...

- *make arrangements of 2 through 5 items using all the items (B)
- *define and identify subsets of a given set of objects (B)
- *make subsets of 1 or 2 objects from sets of 3 or 4 concrete objects (B)

2.12.05 Ratio - 2:1 correspondence

The student will be able to...

- *use two-to-one correspondence to develop the concept of ratio (B)
- *use a chart to show ratio (B)

3.12.01 Bar graphs

The student will be able to...

- *read and interpret a bar graph (A)
- *construct a bar graph (A)

3.12.02 Probability

The student will be able to...

- *estimate (predict) the outcomes of events, and compare the results with the predictions (A)
- *predict the outcome of unequally likely events (A)

3.12.03 Arrangements - tree diagrams

The student will be able to...

- *list and count the number of possible arrangements in a set of objects (A)
- *make a tree diagram to display arrangements of given data (A)

4.12.01 Graphs

The student will be able to...

- *make frequency tables from tallied data (A)
- *graph data drawn from newspapers, magazines, polls, and appropriate activities (A)
- *read and interpret information from a line graph (B)
- *construct a line graph from a table of data (B)

4.12.02 Statistics

The student will be able to...

- *find the mean (average) from given data (A)
- *solve word problems involving finding the averages of given numbers (A)
- *define the range as the difference between the largest and the smallest numbers in a set (A)
- *identify the mode as the number that occurs most frequently (B)

4.12.03 Probability

The student will be able to...

- *determine probabilities by comparing favorable outcomes with total possible outcomes and express as fractions (A)
- *determine probabilities where the outcomes are not equally likely and express as fractions (A)
- *predict the outcomes of probability experiments (A)

4.12.04 Arrangements

The student will be able to...

- *identify all the possible arrangements for a group of objects (A)
- *solve word problems related to skills learned in this module (A)

5.12.01 Graphs

The student will be able to...

- *construct and interpret a double bar graph (B)
- *choose the appropriate graph to show information (B)

5.12.02 Statistics -mean and median

The student will be able to...

- *compute the arithmetic mean (average) to the nearest whole number (A)
- *recognize that the median is the middle score when the group of scores is arranged according to size (limit to odd number of scores) (B)
- *analyze collected data according to range and means (A)

5.12.03 Circle graphs

The student will be able to...

- *interpret circle graphs when the sectors represent fractional numbers with unlike denominators (A)
- *construct circle graphs where the sectors represent fractional numbers with unlike denominators (A)

5.12.04 Probability

The student will be able to...

- *identify events that are impossible, certain, and that sometimes occur (A)
- *express probability as a fraction (A)
- *recognize that some experiments have outcomes that are not equally likely (A)
- *represent and count the number of elements in the sample space of an experiment (A)
- *conduct probability experiments using an increasing number of trials so that the results become more reliable (A)

5.12.05 Algebra - ordered number pairs

The student will be able to...

- *plot an ordered pair on a graph (A)
- *solve equations with two variables to find ordered pairs and graph the results (B)

6.12.01 Graphs

The student will be able to...

- *interpret and construct circle graphs from a table of data (A)
- *construct and compare bar, line and circle graphs and note the advantages and disadvantages of each (A)

6.12.02 Statistics

The student will be able to...

- *find the mode of a given set of data (A)
- *find the median of a given set of data (A)
- *identify the mean, median and mode after involvement in an extended recording project (A)

6.12.03 Compound Probability

The student will be able to...

- *determine the probability of one outcome from a sample space (A)
- *determine the probability of two independent events occurring together from a sample space (A)

6.12.04 Coordinate systems

The student will be able to...

- *locate, read, and interpret coordiantes on a graph (C)
- *graph points whose coordinates include positive and negative numbers using all 4 quadrants (C)

6.12.05 Permutations and combinations

The student will be able to...

- *count the number of permutations that can be made with a set of objects (A)
- *use the multiplication principle to determine the number of permutations that can be made with a set of objects
- *count the number of groups when making combinations (A)

7.12.01 Probability

The student will be able to...

- *construct a sample space for various experiments (A)
- *read and use the expression P(A) to represent the probability of event A (A)
- *write the probabilities involved with simple experiments: flipping a coin, rolling a die (A)

7.12.02 Probability with a spinner

The student will be able to...

- *state that the sum of the probabilities in a given experiment is always 1 (A)
- *state and use the meaning of equally likely events (A)
- *state and use the meaning of complementary events to their probabilities (A)

7.12.03 Multiple graphs

The student will be able to...

- *construct, read and interpret a multiple bar graph (involving a two bar comparison) (B)
- *read and interpret a multiple line graph (involving a two line comparison) (B)
- *construct, read and interpret a circle graph (involving fractional or percent comparisons) (B)

7.12.04 Frequency tables

The student will be able to...

- *obtain the mode from the frequency table (A)
- *recognize how changes in data affect the mean, median and mode (A)

7.12.05 Compound probability

The student will be able to...

- *determine the probability of two independent events occurring together by using a sample space (B)
- *determine the probability of either of two outcomes occurring by using a sample space (B)

8.12.01 Multiple graphs

The student will be able to...

- *construct, read and interpret a multiple bar graph (involving a two bar comparison) (A)
- *construct, read and interpret a multiple line graph (involving a two line comparison) (A)
- *construct, read and interpret a circle graph (involving fractional or percent comparisons) (A)

8.12.02 Statistics

The student will be able to...

- *read, interpret and make a histogram (A)
- *make a frequency polygon from a histogram (A)

8.12.03 Comparison of permutations and combinations

The student will be able to...

- *determine a permutation of n things taken r at a time where $r \leq n$ (A)
- *compare the number of arrangements in a permutation with the number of arrangements in a combination (A)

8.12.04 Compound probability

The student will be able to...

- *determine the probability of two independent events occurring together by using a sample space and a formula (A)
- *determine the probability of either of two outcomes occurring by using a sample space and a formula (A)
- *determine the probability of more than two independent outcomes occurring at the same time using a formula (B)
- *determine the probability of exactly, at least or at most r successes in n trials by experimentation (B)

PROBLEM SOLVING

Topic 13

0.13.01 Problem solving: language

The student will be able to...

- *locate and describe objects in the classroom
- *locate and describe objects within a picture
- *make and explain inferences based upon information obtained from a picture

0.13.02 Problem solving: one-to-one correspondence

The student will be able to...

- *match pictures and objects
- *solve simple problems using one-to-one correspondence

1.13.01 Use drawings to represent objects

The student will be able to...

- *fill in the missing part of a picture (A)
- *make drawings to represent objects in a picture (A)

1.13.02 Develop a mathematical language through questions

The student will be able to...

- *identify numerical information within a picture (A)
- *distinguish between a fact and a question (A)
- *develop questions based upon information obtained from a picture (A)

1.13.03 Use tables to develop question skills

The student will be able to...

- *collect and record data on a table (A)
- *develop questions based upon information obtained from a table (A)

1.13.04 Solve story problems

The student will be able to...

- *solve story problems through dramatization(A)
- *solve story problems using a number line(B)

1.13.05 Story problems/number sentences

The student will be able to...

*translate a story problem into a number sentence (A)

*create a story problem given a number fact through 9 (A)

1.13.06 Problem solving: patterns

The student will be able to...

*identify the rule for a non-numerical pattern, given the pattern (A)

*identify the rule for a numerical pattern, given the pattern (A)

*identify the patterns for sums and differences using concrete materials and tables (B)

2.13.01 Locate information

The student will be able to...

*locate information in the environment in order to solve simple problems (A)

*locate information in a picture in order to solve simple problems (A)

*locate information on a table or chart in order to solve simple problems (A)

2.13.02 Guess and check

The student will be able to...

*check guesses against problem conditions in non-numerical problems (A)

*check guesses against problem conditions in numerical problems (A)

2.13.03 Use data

The student will be able to...

*answer questions based upon a set of data (A)

*create questions based upon a set of data (A)

*make inferences based upon a set of data (A)

2.13.04 Patterns

The student will be able to...

*create and explain a pattern using concrete material (A)

*complete a picture pattern and state the rule (A)

*complete a numerical pattern and state the rule (A)

2.13.05 Missing information

The student will be able to...

- *determine whether sufficient information exists to solve a problem (A)
- *supply missing information to solve a problem (A)

3.13.01 Guess and check

The student will be able to...

- *make and verify a guess (A)
- *check his/her guess against problem conditions (A)

3.13.02 Locate information

The student will be able to...

- *locate pertinent information within a chart/table to solve problems (A)
- *locate pertinent information within a paragraph to solve problems (A)

3.13.03 Patterns

The student will be able to...

- *complete a numerical pattern and state the rule for the pattern (A)
- *arrange numbers to form a pattern and state the rule for the pattern (A)

4.13.01 Problem solving: use tables and graphs to locate information

The student will be able to...

- *locate information on a table/chart and solve problems using this information (A)
- *locate information on a graph and solve problems using this information (A)

4.13.02 Problem solving - use number sentences to solve problems

The student will be able to...

- *translate a one step story problem into an open number sentence and solve (A)
- *choose and sequence operations to solve a two step story problem (A)
- *create a story problem based upon a solution (B)

4.13.03 Problem solving - guess and check

The student will be able to...

- *check guesses against problem conditions involving place value problems (A)
- *check guesses against problem conditions involving addition and subtraction problems (A)
- *check guesses against problem conditions involving multiplication and division problems (A)

4.13.04 Problem solving - patterns

The student will be able to...

- *locate patterns on a calendar (A)
- *locate patterns on a chart/table (A)

5.13.01 Sets

The student will be able to...

- *identify the set which is the union of two finite sets using Venn diagrams (A)
- *identify the set which is the intersection of the two finite sets using Venn diagrams (A)
- *use a Venn diagram to show that the members of subset A are also members of the universal set B (A)

5.13.02 Use prior knowledge to solve problems

The student will be able to...

- *solve problems based upon a model using past knowledge (A)
- *solve story problems using past knowledge (A)

5.13.03 Problem solving - missing information

The student will be able to...

- *determine whether sufficient information exists to solve a problem (A)
- *supply reasonable missing information to help solve a problem (A)

5.13.04 Problem solving - lists and tables

The student will be able to...

- *solve problems by arranging information in a list (A)
- *solve problems by arranging information in a table (A)

5.13.05 Problem solving - guess and check

The student will be able to...

- *check guesses against problem conditions in addition problems (A)
- *check guesses against problem conditions in multiplication problems (A)
- *check guesses against problem conditions (A)

6.13.01 Organize data to perceive patterns

The student will be able to...

- *recognize patterns to solve problems (A)
- *create a simple chart in order to recognize a pettern (A)
- *organize data on a chart to perceive patterns to solve problems (A)

6.13.02 Use number sentences to solve problems with two or more steps

The student will be able to...

- *choose and sequence operations to solve a two step word problem (A)
- *choose and sequence operations to solve a three step word problem (A)

6.13.03 Organize data into tables and charts

The student will be able to...

- *complete a table of missing information (A)
- *organize data into a table in order to solve a problem (A)

6.13.04 Use models to solve problems

The student will be able to...

- *use a model drawing to solve a problem (A)
- *create questions given a model (A)

7.13.01 Solving a simpler but equivalent problem

The student will be able to...

- *use models/chart to simplify a problem (A)
- *solve problems by simplifying the data by finding a pattern (A)

7.13.02 Working backwards

The student will be able to...

- *make a model/chart of data in a problem (A)
- *use a model/chart to solve a problem by working backwards (A)

7.13.03 Guess and check

The student will be able to...

*make and refine a guess about his/her environment (A)

*check guesses about problem conditions (A)

7.13.04 Patterns

The student will be able to...

*identify and extend a numerical pattern (A)

*use patterns to solve problems (A)

8.13.01 Selection by elimination

The student will be able to...

*use information given in a problem to el;iminate all possible solutions except the correct one (A)

*list all possible solutions and then eliminate all but the correct one (A)

8.13.02 Models

The student will be able to...

*make a picture-type model to solve a problem (A)

*select a physical or paper-pencil type model to solve a problem (A)

8.13.03 Tables

The student will be able to...

*make a list to solve a problem (A)

*find a rule from a table (A)

*use a table to generate information not given to solve a problem (A)

LESSON PLANNING IN MATHEMATICS

1. The math teachers should prepare a detailed lesson plan for each lesson.

2. The parts of a lesson plan are: (a) Aim (b) Performance Objectives) (c) Do Now (d) Motivation (e) Development (f) Applications (g) Summary

3. The aim should be stated clearly and simply and written in question form. It should be elicited from the pupils. It need not, however, be elicited at the very beginning of the period.

4. The teacher must recognize what previous learning pupils will need as a foundation on which to base the principle to be taught in the new lesson. The do-now questions should include pre-requisite skills, defintions or procedures needed for the new lesson.

5. The motivation should be brief and to the point. It should lead to a statement of the aim of the lesson and a desire to achieve it. The motivating problem should be simple enough to be solved by the class after the development of the lesson is completed.

6. During the development of the lesson, the teacher should:
   a) maintain control of its pace and direction
   b) avoid dominating it, but promote widespread pupil participation and inter-pupil relations, as much as possible eliciting rather than telling correct answers.
   c) make use of proper questioning techniques, asking questions which are clear and unambiguous, including a number which will stimulate thought and evoke sustained responses by pupils, as well as others which will help pupils recognize shortcomings and errors in some of their answers.
   d) think through the statements that will be made, the questions to be asked and the answers expected from the pupils as the lesson develops.
   e) try to anticipate incorrect answers or unexpected questions to see where they may lead, so that the lesson may not be misdirected. An unexpected wrong answer or a surprising question can easily upset a lesson for the entire class.
   f) include the wording of statements and of questions in the lesson plan.
   g) elicit from pupils complete and accurate statements, suitably and clearly expressed, employing technical vocabulary correctly and intelligently.
   h) check that pupils understand the principle used to obtain correct answers submitted using the lesson.
   i) see that all relevant material is not neglected, and that irrelevant matters do not intrude.
   j) present a sufficient number and variety of situations before calling on pupils to formulate a generalization.

7. The teacher should prepare in advance a statement of the generalization to be reached by pupils in the lesson and of the question he/she will ask in order to elicit this generalization from the class. The statement should be in language that is clear and easily understood by pupils, and mathematically correct. Any technical vocabulary employed should be adequately reviewed. By skillful questioning, the students themselves should be helped by the teacher to formulate the generalization.

8. One of the first applications of the principle learned during the lesson should be to solve the motivating problem previously mentioned. Other applications of this principle should be presented to the class.
When pupils submit correct solutions, the teacher should see that in each the

principle used to obtain them is emphasized. The teacher should prepare in advance a large number--more than he thinks he may need--of carefully graded examples which require the use of this new principle.

9. A summary of what was accomplished during the period should be elicited from the students. The summary is a check upon whether the aim of the lesson has been achieved.

10. The presentation of the new lesson should not be at the expense of reviewing the homework. The teacher must see that there is time for both.

11. It is also preferable that a teacher plan to cover more than can be covered in a period rather than find a void in the lesson for even a few minutes.

SIGNIFICANT ASPECTS OF A MATHEMATICS LESSON

1. "Do Now" (Warm-Up) or (Seatwork)
   a. Should be related to the new lesson, not just an arbitrary exercise i.e. should include pre-requisite skills, definitions, or procedures needed for the new lesson of the day.
   b. Should be written on the board at the beginning of the lesson.
   c. Should serve as a transition between previous learnings and new.
   d. Should take no longer than five minutes to review.
   e. Should be short and attainable by students within the first few minutes of the period.
   f. Can be used to provide writing experience where appropriate by asking students to write definitions, procedures, explanations, etc.
   g. Does not have to be used in a math lesson where the lesson begins immediately.

2. Motivation
   a. Should create a need to know ("in order to solve this problem we must learn this new technique...") or a desire to know ("Why does this card trick work?")
   b. May lead from a problem or apparent paradox
   c. Is short
   d. Yields the aim
   e. Is referred to in the summary

3. Aim
   a. Should be in the form of a question.
   b. Can be easily measured for achievability.
   c. Should be written on the board and left for the entire period.

4. Homework
   a. Should be cyclical (i.e., examples from a month ago, week ago, day ago)
   b. Should be reasonable in length
   c. Should take at most 10-12 minutes to review, preferably less.
      1. Student should "carry" review.
      2. Solutions and questions should be written out on students' work.
      3. Board responsibilities should be assigned in advance.
      4. Can employ visual devices (overhead, colored chalk, etc.) to save time and promote clarity.
   d. Should be numbered.
   e. Should be assigned at the start of the period and written on the same area of the board or be included in a homework assignment sheet.
   f. Should include RCT math or regents review questions.
   g. Should be solved by teachers in their lesson plans.
   h. Should be checked or collected.
   i Can be referred to appropriately in the new lesson.
   j. Provides pre-requisite skills for the new lesson.
   k. May be reviewed at the beginning or the end of the period.
   l. Should include a reading assignment.

5. Development
   a. Concepts should be taught along with techniques i.e., the "why" along with the "how"
   b. One new thought should be developed at a time in the sequence of logical analysis.
   c. The problems used as models should be well chosen and graded in diffculty. i.e., they yield the desired concepts and skills to be studied.
   d. The joy and beauty of the mathematics should be revealed.
   e. Concrete models, diagrams and other visual devices should be employed.

f. The board work should be detailed, neat and written from left to right on the board.
g. Students should share a mutual discovery with the teacher.

6. Questioning
   a. Skillful questioning serves to direct activity:
      - to provoke discussion which will extend knowledge and awareness
      - to stimulate thinking
      - to measure accomplishment by testing information and understanding
      - to seek evidence of reflective thinking and judgments based on proof.
   b. Skillful questioning should keep the student more alert, more critical, more active. Therefore, frame questions of graded difficulty in order to maintain interest, and appeal to the varying abilities of students. Distribute questions, calling on non-volunteers as well as volunteers. Handle the erroneous answers of the weaker students without belittling their attempts. Encourage topical, sustained answers.
   c. To stimulate critical thinking, questions should evoke sustained answers. Therefore, it is poor practice to ask too many questions or to use only factual questions.
   d. After asking a difficult question, it is wise to pause before calling on a student. Such a pause will provide time for thought and discourage the quick, glib answer. The pause also provides time for students who think more slowly to react.
   e. Once a student begins a developed answer, permit him to finish without interrupting him.
   f. Since at all times responses should be directed to the class rather than to the teacher, other class members are free to challenge the accuracy of these responses. (Such automatic procedures result, of course, from constant, patient training in the early weeks of the term and from reminders given constantly throughout the term.) Where an individual--teacher--individual recitation is used, general student participation is impossible, and lack of respect for others as well as inattention (day-dreaming, talking, etc.) will result. The teacher has to keep the group in the picture even when dealing with an individual problem.

   g. <u>Criteria for judging questions asked in the classroom</u>

      1. They should be simple and direct in language.
      2. They should be specific in directing attention to the particular problem under consideration.
      3. They should follow a planned order which develops the train of thought in a logical sequence.
      4. They should place the burden of thinking upon the student (and, therefore, should be relatively few in number).
      5. They should be so framed that the answer may <u>not</u> be a simple 'yes' or 'no'.
      6. There should be included in the lesson questions which require students to draw conclusions.
      7. Questions should be adapted to the varying levels of ability in the class.
      8. They should be sufficiently difficult to arouse effort, yet not so difficult as to preclude a response.
      9. There should be a proper balance between fact and thought questions.
      10. Questions should not be repeated. (If you must have one repeated, have a student repeat it).

7. Applications and Drill Problems
   a. Should be graded from simple problems to complex
   b. Should be reviewed on the board by students in the class
   c. Should provide enrichment for talented students
   d. Should be pre-planned (and solved completely in teacher's lesson plan).

8. Summary
   a. Should be elicited from students
   b. Often yields a "recipe" type of problem-solving procedure. Should include key concepts as well.
   c. Can be generated from a "tour" of board work from left to right.
   d. Should refer to the aim.
   e. A medial summary, elicited from the class, should be provided prior to application and/or drill.
   f. A final summary elicited from the class, should be provided after the application problems.

9. Miscellaneous
   a. Teacher should circulate around the room monitoring notebooks and assisting students with individual problems.
   b. Teachers' board work should serve as a model. They should use a ruler, be neat and write out full sentences. Diagrams should be easy to read.
   c. Teachers should have worked out solutions to all homework problems and problems used during the period.
   d. All problems used should not be test oriented.
   e. Timing of a typical lesson:
      5 minutes--"Do Now"
      5-10 minutes--Homework review
      12-20 minutes--Development
      7 minutes--Drill and/or applications.
      2 minutes--Summary

TECHNIQUES OF MOTIVATION

1. Indicate a void in student's knowledge.
2. Show a sequential development.
3. Present a challenge.
4. Indicate the usefullness of a topic.
5. Use recreational mathematics.
6. Tell a pertinent story.
7. Get students actively involved in justifying mathematical curiosities.
8. Use teacher-made or commercially prepared materials.

Posamentier, Alfred & Stepelman, Jay, Teaching Secondary School Mathematics (Charles Merrill Publishing Company, 1981).

THE USE OF THE "DO NOW" IN A MATHEMATICS LESSON

I. Why do we give "do now" problems?

A. To review pre-requisite skills, concepts, vocabulary or procedures that will be needed for the new lesson.
B. To provide opportunities for integrating writing in the mathematics class.
C. To set a routine of students beginning their work as soon as they enter the room. This also sets a tone that the entire period is spent on instruction and not irrelevant activities.
D. To provide the teacher with an opportunity to handle clerical matters and yet not interfere with student learning. This is only one of several opportunities that the teacher has to handle such things as taking attendance without having students unoccupied in a learning activity.

II. What are some of the characteristics of a well chosen "do now?"

A. It reviews pre-requisites skills, concepts, vocabulary or procedures needed in the new lesson.
B. It is within the ability of the class to answer.
C. It can be copied and completed by the class in no more than the first five minutes of the period.
D. It can be used as an opportunity to provide mathematics students with writing experiences.

III. What should we avoid when we use a "do now?"

A. Don't allow it to take too much time.
B. Don't allow it to become a review lesson. Review lessons, like all lessons, must be motivated and developed. The "do now" should NOT become the lesson.
C. Don't include the new topic in the "do now" assuming that the students will know how to do it. If the students can answer questions on the new topic, perhaps, you do not need to teach that lesson.
D. The "do now" is not the motivation, although it may contain part of your motivation.
E. The "do now" is not independent from the rest of the lesson. There should be a <u>transition</u> from the "do now" to what follows. It should be referred to at some point in the lesson. If it is not referred to, perhaps it was not needed.
F. Don't use a "do now" if the lesson does not call for it. A lesson can start immediately without a "do now".
G. Don't be redundant. If your homework assignment has reviewed your pre-requisite skills, you need not do it again in a "do now".

SUGGEST AN APPROPRIATE "DO NOW"

FROM MATHEMATICS SCOPE AND SEQUENCE GRADE 3

1. Aim: How can we identify the place value of any digit in a number through 999?

2. Aim: How to complete a numerical pattern and state the rule for the pattern?

3. Aim: How can we create charts/tables to help solve problems?

4. Aim: How can we identify and define the numerator and denominator in a given fraction?

SUGGEST AN APPROPRIATE "DO NOW"

FROM MATHEMATICS SCOPE AND SEQUENCE GRADE 6

1. Aim: How can we find the area of any triangle given the base and the height using the formula $\frac{1}{2} \times b \times h$?

2. Aim: How can we determine the probability of one outcome from a sample space?

3. Aim: How can we express fractions and decimals as an equivalent percent?

4. Aim: How can we rename composite numbers as the product of prime?

FROM LESSON PLANNING TO PERFORMANCE

A. A lesson may not be successful despite the fact that the plan included a worthwhile aim, a challenging motivation and thought-provoking questions. The following are suggestions to help you insure success in the execution of your daily plans:

1. You must be prepared to answer questions that may be raised by the pupils. The answering of such questions may require a knowledge of your subject that goes beyond the content of the textbook. A potent motivation for learning on the part of the pupils can be the scholarship of the teacher.

2. You must be prepared to develop the lesson along a path different from the one you set down in your plan. This way come about because of the answers that are presented in response to your motivation. The class may suggest a better aim than the one you have prepared.

3. You must make provision for the active participation of the greatest number of pupils in each lesson. Do not permit a few students to dominate or by watching demonstrations. To insure the greatest amount of participation it is suggested that:

   a. You establish the rule that hands be raised only when you request volunteered answers or when a pupil wishes to ask a question.

   b. You call on the slow, passive and shy pupils to repeat answers, to answer simple questions, and to evaluate answers.

4. You should make the entire period, not only the motivation time, a stimulating and challenging experience for the class. Suggested procedures to realize this goal are:

   a. Sustain the motivation throughout the period.

   b. Employ questions that stimulate thinking and that produce interaction among the members of the class.

   c. Use demonstrations and experiments that excite the interest of the pupils.

5. Make certain that, throughout the period, understanding and retention are achieved on the part of the pupils. You should recognize that answers may be inaudible, fragmentary, or poorly-phrased; that members of the class may be inattentive at times; and that all students may not comprehend answers or board demonstrations. The following suggestions are given to insure understanding and retention:

   a. Insist that pupils speak in an audible tone to the majority of the class. If you feel that an answer has been inaudible to some members of the class, have it repeated by a student.

   b. If an answer lacks details, if it does not cover all facets of the problems, or if it is totally or partially incorrect: call on other pupils to supplement or to evaluate the answer. If necessary, pose another question that aims at soliciting additional information.

   c. If, after calling on a few pupils, you are unable to elicit the desired response--rephrase the question. If the response is still not forth-

coming, present the answer yourself. Do not waste valuable teaching time in an attempt to secure an answer to any one question.

d. Prepare pupils for the observing of experiments and of board demonstrations. During the execution of experiments and demonstrations ask questions and solicit questions.

e. Call on a student to provide a summary after a number of answers have been presented in response to a particular question.

f. Include drills, medial summaries and final summaries and applications in every lesson.

PRACTICE MAKES PERFECT

A. Many roads lead to Rome. There is no one correct method to teach a particular topic or to achieve a particular aim. The plan used successfully by one teacher with a particular class may not achieve the same results when used by another teacher. You will, as a result of your own planning, discover the most effective methods to be employed in teaching each topic with different classes.

B. More effective teaching on your part will result in better pupil achievement, better class discipline and an easier and a more satisfying teaching day for you. To improve on your lesson plan it is suggested that:

1. You critically evaluate each lesson in terms of the interest of the students and of the learnings that took place. If your plan has not proved effective during the period that you taught it, try to revise it before you teach the same lesson to another class.

2. Secure suggestions from veteran teachers in your department for the teaching of particular lessons.

3. You pre-plan lessons with your supervisor.

4. You observe more experienced teachers.

5. You re-write your lesson plans each year. Classes change; the world changes. An aim or a motivation that is timely in one year may have no appeal a different year.

6. You consult frequently with your subject supervisor.

Practice will not make you a perfect planner or a perfect teacher. There is no such person. What it will do is make you an interested teacher, an inquiring teacher and therefore a better teacher. The secret of your success will lie in your own efforts toward self-improvement.

GRADE 3 LESSON PLAN

AIM: Wnat is meant by an ordered number pair?

PERFORMANCE OBJECTIVES: The student will be able to:
- identify points on a grid by using ordered number pairs
- give directions for getting from one place to another on a grid using horizontal and vertical paths

DO NOW(GETTING STARTED OR WARM-UP)

Write the definition of the following words in a complete sentence:
a. horizontal b. vertical

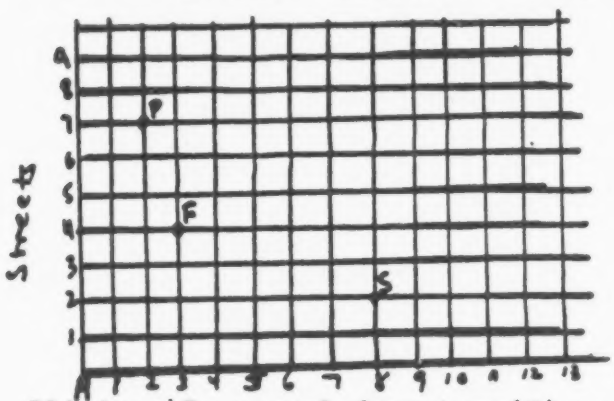
Elicit: Horizontal means going from left to right.

Vertical means going up and down.

MOTIVATION:
Ask the class: When would you use a map?

Elicit: A map is used when locating a place.

DEVELOPMENT:
Ask the class to use the following map and describe how they would travel from their home(H) to the park(P)
from their home(H) to the school(S)



Elicit:a)To travel from home(H) to the park(P) we would move
2 blocks horizontally and 7 blocks vertically or move
7 blocks vertically and 2 blocks horizontally.
b)To travel from home(H) to the school(S) we would move
8 blocks horizontally and 2 blocks vertically or move
2 blocks vertically and 8 blocks horizontally.

Explain to the class that if we want to give directions without having to use the words horizontally or vertically, we need to determine an order so that one knows which is first. Therefore we will move horizontally first, then vertically.

Explain that mathematicians are very lazy and also want to write as little as possible, so that we can write
(2,7) to mean moving 3 blocks horizontally and 7 blocks vertica
Therefore P is located at (2,7)

(8,2) to mean moving 8 blocks horizontally and 2 blocks vertical
Therefore S is located at (8,2)

Explain that this is true only if we start at H which is (0,0) which means no movement horizontally and no movement vertically.

Explain to the class that this map is called a grid since it has both horizontal and vertical lines indicated(avenues and streets are used on this map).

Place another point on the grid to indicate a friend's house(call it F)

Ask the class to describe how to travel from home(H) to the friend's house (F) and how to identify point F.

Elicit: To travel from H to F, move 3 blocks horizontally and 4 blocks vertically from H, and point F is written as (3,4).

Ask the class: Explain whether (3,4) is the same as (4,3)

Elicit: (3,4) is NOT the same as (4,3) since (3,4) means moving 3 blocks horizontally and 4 blocks vertically and (4,3) means moving 4 blocks horizontally and 3 blocks vertically.

Explain that because order is important, we call (3,4) an ordered number pair

APPLICATIONS:

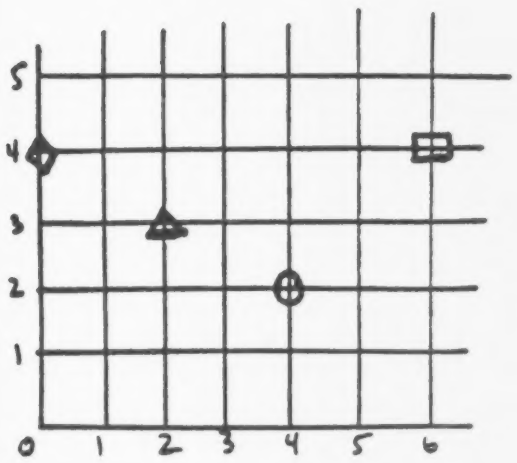
Use the same map(grid) and ask students to locate the following ordered number pairs:

a) (2,6) b) (5,7) c) (0,8)

Ask students to explain how they traveled(or moved) to get to each of these points. Write the description of the movement on the chalkboard.

e.g.
(2,6) means moving 2 blocks(spaces) horizontally and 6 blocks vertically

Use the following grid and answer the following questions:



a) What is the number pair for the circle?

b) What figure has the number pair (2,3)?

c) What is the number pair for the diamond?

d) What figure has the number pair (5,4)?

FINAL SUMMARY:

a) When would we use a number pair?
   Elicit: A number pair is used to give the location of a point on a grid(graph)?

b) Describe the direction of movement for a number pair such as (5,2).
   Elicit: The first number (5) means moving 5 blocks horizontally and the second number(2) means moving 2 blocks vertically.

c) Why do we use the term ordered number pair?
   Elicit: Number pairs are ordered so that we don't think (5,2) is the same as (2,5).

ENRICHMENT ACTIVITY:

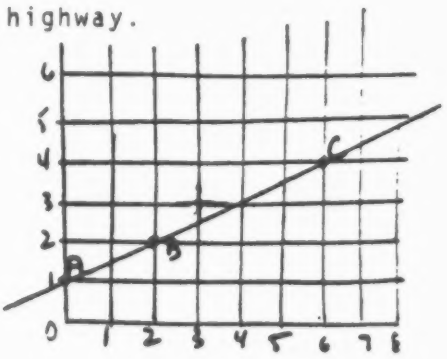
The line on the map below is a highway.

Write the ordered pairs for the points on the highway.

i.e. points A,B, and C.

You are at (4,3). Are you on the highway? Explain your answer.

You are at (8,4). Are you on the highway? Explain your answer.



SAMPLE GRADE 4 LESSON PLAN

AIM: How can we change improper fractions to mixed numbers?

PERFORMANCE OBJECTIVES: The student will be able to:
- identify an improper fraction
- identify a mixed number
- change an improper fraction to a mixed number
- state the procedure used for changing an improper fraction to a mixed number

DO NOW(WARM-UP):

1. Write the fraction having a numerator of 2 and a denominator of 5.

2. In the fraction $\frac{3}{10}$, what is the meaning of the 10 and what is the meaning of the 3?

Elicit:

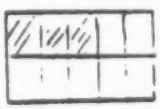
(1) The fraction is $\frac{2}{5}$. Write on board: $\frac{2}{5}$ → numerator
$\frac{2}{5}$ → denominator

(2) In the fraction $\frac{3}{10}$, the 10 means that the object is divided into 10 equal parts and the 3 means that you are using only 3 of those equal parts.

Write on board:
$\frac{3}{10}$ -the number of those equal parts being taken
$\frac{3}{10}$ -the number of equal parts the object is divided

Ask the class to show $\frac{3}{10}$ using the rectangle.

Put diagram on board and have a student come to the board to show $\frac{3}{10}$.



Write on board:
Shaded part = $\frac{3}{10}$

MOTIVATION:

Present the following problem orally to the class and write on board.

Each apple pie at this school's cake sale is cut into eighths. How many pies must we buy if the 10 students in this class each want a piece of apple pie?

Ask the class: What information is given to us in this problem?
Elicit: There are 10 students in this class who want a piece of pie.
Each pie is divided into 8 equal pieces.

Ask the class: What do we want to find?
Elicit: We want to find how many pies are needed if we are taking $\frac{10}{8}$ pieces of pie.

(Note: If students have difficulty writing $\frac{10}{8}$, refer them back to the do now problems)

Explain to the class that $\frac{10}{8}$ is called an <u>improper fraction</u> since the numerator of the fraction is larger than the denominator of the fraction. Write the definition of an improper fraction on the board.

Ask the class: How can we find the number of pies needed for $\frac{10}{8}$ pieces of pie?

Elicit: Using a diagram. Have student come to the board to explain how a diagram may be used to solve the problem.

Elicit since $\frac{8}{8}$ equal one whole pie, then $\frac{10}{8}$ means $1 + \frac{2}{8}$.

Ask the class: What is the answer to the problem.
Elicit: We need to buy 1 whole pie and $\frac{2}{8}$ of another pie. & write on board.

Explain to the class that $1 + \frac{2}{8}$ may be written as $1\frac{2}{8}$ and that $1\frac{2}{8} = 1 + \frac{2}{8}$ = a mixed number. Write this also on board.

Ask the class: How can we decide if a number is a mixed number?
Elicit: A mixed number is the sum of a whole number and a fraction.

Refer the class back to the fraction $\frac{10}{8}$.

Ask the class: Why did we call this fraction an improper fraction?
Elicit: Since the numerator of $\frac{10}{8}$ is larger than the denominator, the fraction is called an improper fraction.

Ask the class: How else may we write $\frac{10}{8}$?

Elicit: $\frac{10}{8}$ = 10 divided by 8 = $10 \div 8 = 8\overline{)10}$ Write these statements on board.

Ask the class to divide. Elicit: $8\overline{)10}$ $\begin{array}{r} 1\text{R}2 \\ \underline{8} \\ 2 \end{array}$ or $8\overline{)10}$ $\begin{array}{r} 1\frac{2}{8} \\ \underline{8} \\ 2 \end{array}$ Write models on board.

Therefore $\frac{10}{8} = 1 + \frac{2}{8} = 1\frac{2}{8}$

<u>MEDIAL SUMMARY</u>: Ask the class How can we change an improper fraction to a mixed number?

Elicit by having students express in words what is seen on the board in the model problem.

Write on board as you elicit the steps under the question "How can we change an improper fraction to a mixed number?

1. Divide the numerator by the denominator.
2. Write the whole number part of the answer(quotient)
3. Write the remainder over the divisor as the fraction part.

APPLICATIONS:

1. Which of the following is(are) improper fractions? Explain why.
   a. $\frac{2}{3}$ b. 9 c. $\frac{14}{7}$ d. $\frac{18}{5}$

2. Write the number of parts shaded in two different ways.

Elicit and write on board:
shaded part = $\frac{13}{4}$ or $3\frac{1}{4}$

Ask class to name their answers.
Elicit and write on board: 3 mixed numbe
$\frac{13}{4}$ improper fracti

Have students go to the board to explain their work and write their work for problems 1 and 2 above.

3. Change each of the following to a mixed number.
   a. $\frac{19}{5}$ b. $\frac{13}{7}$

Have two students come to the board to write and explain their work.

FINAL SUMMARY: Ask the class: What is an improper fraction? a mixed n
What is the procedure used to change an improper fraction to a mixed number?

SAMPLE GRADE 5 LESSON PLAN

AIM: How can we find the perimeter of polygons?

PERFORMANCE OBJECTIVES: The student will be able to:
- state and write the definition of perimeter
- compute the perimeter of polygons

DO NOW(WARM-UP):

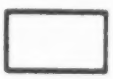
Define and draw a diagram for each of the following terms:
a. rectangle b. square c. trapezoid

Elicit: diagrams for each term and have students give complete sentences for definitions. Write definitions on board under each diagram.

MOTIVATION:

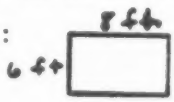
Present the following problem orally to the class and write on board:
A fence is to be placed around a small rectangular garden whose dimensions are 6 feet by 8 feet. How much fencing will be needed to surround this garden?

DEVELOPMENT:
Ask the class to sketch a diagram for this problem:
Elicit:



Ask the class to indicate in the diagram the given information from the problem.

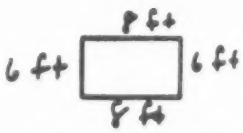
Elicit:



Ask: What is meant by dimensions of rectangle?
Elicit: Dimensions of rectangle are its length and width.

Ask the class to explain what the problem asks them to find.
Elicit: We need to find how many feet of fencing is used to surround the garden.

Ask the class: Do we have enough information to solve the problem?
Elicit: Since the opposite sides of a rectangle are equal, we can indicate the lengths of the remaining sides.



Ask the class: How can we determine the amount of fencing needed to surround the garden?

Elicit: Since the word surround means around the outside of the rectangle, we can find the amount of fencing needed by adding the lengths of the four sides of the rectangle.

Elicit and write on board:
Amount of fencing = 8 ft + 6 ft + 8 ft + 6 ft = 28 ft

Ask the class in what other ways the amount of fencing can be determined.

Elicit: $8 + 6 = 14$ or $8 + 8 + 6 + 6$

$2(14) = 28$ $= 2(8) + 2(6)$

$= 16 + 12$

$= 28$

Ask the class to answer the problem.
Elicit & write: 28 ft of fencing is needed to surround the garden

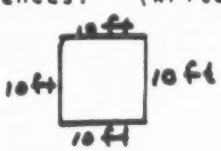
Explain to the class that we call the perimeter of a rectangle the sum of the lengths of its four sides. (write definition of board)

Ask class what would we mean by the perimeter of a triangle.
Elicit: The perimeter of a triangle is the sum of the lengths of its 3 sides.

Ask the class to find the amount of fencing needed to surround a garden in the shape of a square with each of its sides of length 10 ft.

Elicit the diagram needed, the given information, what the problem is asking them to find, the solution and the answer in complete sentences. (write on board)

Diagram:



Given: The length of each side is 10 ft.

Find: How many feet of fencing is needed to surround the garden. We need to find the perimeter of the square.

Solution: $p = 10 + 10 + 10 + 10 = 40$ ft.

or

$p = 2(10 + 10)$ or $p = 4(10)$ or $p = 2(10) + 2(10)$

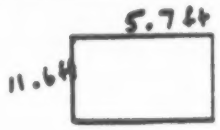
$= 2(20)$ $= 40$ ft $= 20 + 20$

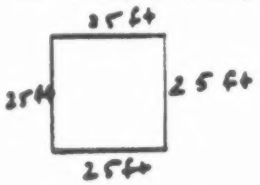
$= 40$ ft $= 40$ ft

Answer: 40 feet of fencing is needed to surround the garden.

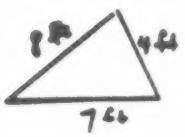
APPLICATIONS:

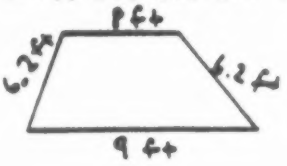
Give students problem 1 and 2 to work at their seats. Then have two students come to the board to write and explain their work to the class.





Give students problem 3 and 4 to work at their seats. Then have two students come to the board to write and explain their work to the class





FINAL SUMMARY:

Ask the class to explain the method for finding the perimeter of a rectangle, for finding the perimeter of a square, for finding the perimeter of a triangle.

SAMPLE GRADE 6 LESSON PLAN

AIM: How can we subtract mixed numbers from whole numbers?

Performance Objectives: The student will be able to:
- identify and describe a mixed number
- subtract a mixed number from a whole number with simplification
- express 1 in equivalent forms

DO NOW(Warm-Up):

1. Which of the following is a mixed number? Explain your answer.

   a. $\frac{1}{2}$ b. 7 c. $8\frac{3}{5}$ d. $\frac{19}{2}$

2. Simplify

   $\frac{2}{2} = ?$ $\frac{4}{4} = ?$ $\frac{8}{8} = ?$

3. Write 1 as a equivalent fraction having 6 as a denominator.

Elicit:

1. A mixed number is the sum of a whole number and proper fraction. Therefore, $8\frac{3}{5}$ and $\frac{19}{2} = 9\frac{1}{2}$ are examples of mixed numbers.

   Write on board:

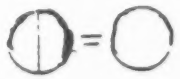
   $8\frac{3}{5} = 8 + \frac{3}{5}$; $9\frac{1}{2} = 9 + \frac{1}{2}$

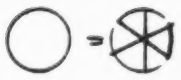
2. $\frac{2}{2} = 1$; $\frac{4}{4} = 1$; $\frac{8}{8} = 1$

   Ask students to explain their answers. (They may use a circle to show these results or other diagrams.)

   Place appropriate diagram on board under the statement.

   e.g. $\frac{2}{2} = 1$



3. $1 = \frac{6}{6}$ 

MOTIVATION

Present the following problem to the class orally and write on chalkboard:

Mary Baked 7 apples pies for a cake sale. She sold $2\frac{1}{2}$ pies. How many apple pies did she have left?

Have a student read the problem aloud to the class.

Ask students: What information is given to us in this problem?
Elicit: Mary baked 7 pies and sold $2\frac{1}{2}$ pies?

Ask students: What do we want to find?
Elicit: How many apple pies Mary had left after selling $2\frac{1}{2}$ pies

Ask students: What can be said about the number of pies left?
Elicit: The number of pies left must be less than 7.
or
If we estimate the number of pies sold to 3 to make the problem simpler, then there are 4 pies left.(since 7-3 = 4)

Ask students: Describe a plan that may be used to solve this problem.
Elicit: A diagram ○○○○○○○ - ○○○ or A number sentence $7 - 2\frac{1}{2} = ?$

Have students explain why the operation of subtraction used to solve this problem.

Ask: What do you think we are going to learn about today?
Elicit: How can we subtract mixed numbers from whole numbers.

DEVELOPMENT:

Ask the class to solve:
$$\begin{array}{r} 51 \\ -\ 29 \\ \hline \end{array}$$

Have a student explain how to solve the problem.
Elicit: In order to subtract, we have to exchange 1 ten for 10 ones.
Have a student come to the board to solve the problem:
On board:
$$\begin{array}{r} 41 \\ 51 \\ -\ 29 \\ \hline 22 \end{array}$$

Ask the class to solve:
$$\begin{array}{r} 5\frac{2}{3} \\ -\ 1\frac{1}{3} \\ \hline \end{array}$$

Have a student come to the board to do the problem and explain his/her work. Elicit: In order to subtract, we subtract the fractional parts & then subtract the whole number parts of the mixed numbers

On board:
$$\begin{array}{r} 5\frac{2}{3} \\ -\ 1\frac{1}{3} \\ \hline 4\frac{1}{3} \end{array}$$

Ask class what type of numbers are being subtracted.
Elicit: This problem involves subtracting a mixed number from a mixed number.

How can we check our answer:
Elicit and write check on board.

How can we use what we just did to solve:
$$\begin{array}{r} 7 \\ -\ 2\frac{1}{2} \\ \hline \end{array}$$

Elicit: In order to subtract, we need to exchange 1 for $\frac{2}{2}$ and rename

Ask how they knew to use $\frac{2}{2}$ and not $\frac{3}{3}$?

On board:

$$\begin{array}{r} 7 = 6\frac{2}{2} \\ -\ 2\frac{1}{2} = 2\frac{1}{2} \\ \hline 4\frac{1}{2} \end{array}$$

If needed elicit and write on board:

$$\begin{aligned} 7 &= 6 + 1 \\ &= 6 + \frac{2}{2} \\ &= 6\frac{2}{2} \end{aligned}$$

Ask how can we check our work.

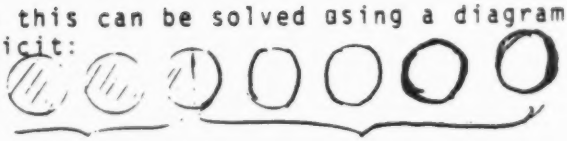
Elicit: Check

$$\begin{array}{r} 4\frac{1}{2} \\ +\ 2\frac{1}{2} \\ \hline 6\frac{2}{2} \end{array}$$

$$\begin{aligned} 6\frac{2}{2} &= 6 + \frac{2}{2} \\ &= 6 + 1 \\ &= 7 \end{aligned}$$

Ask how this can be solved using a diagram.

Elicit:



amount sold — amount of pies left

Ask students to answer the motivation problem.

Elicit: Mary had $4\frac{1}{2}$ pies left from the cake sale.

MEDIAL SUMMARY:

How can we rename a whole number to a mixed number?

APPLICATIONS:

Ask students to refer to the model on the board when solving the following problems:

1. $\begin{array}{r} 10 \\ -\ 4\frac{2}{3} \\ \hline \end{array}$

2. $\begin{array}{r} 12 \\ -\ 9\frac{1}{4} \\ \hline \end{array}$

Have students go to the board to write and explain their procedures and solutions to these problems.

If time, give problems 3 and 4 and have students come to the board to write and explain their work.

3. $\begin{array}{r} 50 \\ -\ 18\frac{5}{6} \\ \hline \end{array}$

4. $\begin{array}{r} 100 \\ -\ 18\frac{2}{5} \\ \hline \end{array}$

FINAL SUMMARY:

Ask: Explain the procedure to subtract a mixed number from a whole number.

Situations for Discussion by New Teachers: Lesson Planning

1. You discover, after the first two weeks of school, that you are already two lessons behind a more experienced teacher who is teaching the same course. What do you do?

2. You find, after the first 25 minutes of a 40 minute lesson, that you have reached the end of your lesson plan. What do you do?

THE ART OF QUESTIONING

Effective questioning is the tool with which teachers develop logical thought, stimulate the imagination, bridge the gap between previous learning and new learning and give direction to the learning process. When skillfully phrased and strategically used, questions become one of the teacher's most flexible instructional tools. On the other hand, poor questions and poor questioning techniques interfere with learning by creating confusion, discouraging pupil involvement and inhibiting the sequential development of concepts.

General characteristics of effective questions:

- They are kept short.
- They are simply phrased and direct in language.
- They are logically sequenced, the response to one question leading into the next, with each question calling for only one or two points in the chain of development.
- They are designed to require pupils to think, rather than respond instantly with an association. Whenever possible, they elicit a sustained response rather than a one- or two-word answer.
- They are frequently personalized to grasp the pupils' attention: "How would you explain to a classmate who was absent yesterday our method for changing common fractions to decimal fractions?"

Twelve Types of Questions to Avoid: A Dirty Dozen

Teachers sometimes develop ineffective questioning habits that are difficult to break unless conscious examination in precisely what kinds of questions they have been asking takes place. Ineffective questions come in many varieties, and are ineffective for various reasons. We have all heard them, and have perhaps used some of them. Here is a brief compilation of some of them:

(1) **The Multiple or Overlaid Question:**

Example: What is the number above the fraction bar called, and what is the number below the fraction bar called? How do we reduce fractions?

Criticism: By asking for more than one response, the teacher is limiting participation to only those pupils who know all of them. Many more pupils might be able to respond if a single question were asked at a time.

Suggestion: Ask the three questions separately, waiting until one question is answered before going on to the next.

(2) **The Vague or Ambiguous Question:**

Example: How about rectangles?

Criticism: The question doesn't direct pupil thinking in any particular direction. A multitude of irrelevant responses can be made.

Suggestion: What kind of angles do the sides of a rectangle form?

(3) **The Choral Question:**

Example: Were we right to multiply by 100?
(Y-e-e-e-e-s!)

Criticism: Pupils are not made to feel personally responsible for the accuracy of their responses. They are encouraged to hide behind the anonymity provided by choral response. Different responses to the same question are uttered together, creating confusion for individual pupils who may not know which one is being accepted as correct.

Suggestion: Do you think we were right to multiply by 100, ... Joseph?

(4) **The Leading Question:**

Example: There are 6 fours in 24, aren't there?

Criticism: A pupil who is not sure will promptly agree with the teacher without thinking about his or her response. The pupil has been invited merely to nod his or her head in acquiescence.

Suggestion: How many fours are there is 24, ... Sherry?

(5) **The Tugging Question:**

Example: The area of a rectangle is found by multiplying the length by the, ___ ...? W___, w___. Come on now; it starts with "W".

Criticism: The question reduces the discussion to a guessing game, often not even based on the subject matter but on the recall of words by virtue of their sounds.

Suggestion: How is the area of a rectangle found? OR: What two measures do we multiply to get the area of a rectangle?

(6) **The Teacher-Oriented Question:**

Example: Who can tell me how many places to move the decimal point when multiplying by 1000?

Criticism: The question contributes to a feeling that the class is organized around the teacher, and that what is most important is satisfying the teacher's requirements. It does not contribute to a group feeling of a class working together to learn.

Suggestion: Who can tell us how many places to move the decimal point when multiplying by 1000?

(7) **The Whip-Lash Question:**

Example: The unit of measure that is equal to 1000 meters is called the what?

Criticism: The question is not perceived as a question by the pupils since it does not begin with an interrogative. It sounds as though the teacher is making a declarative statement. Only at the last word do pupils learn that they are being asked for a response, and thus have no time in which to think about it.

Suggestion: Include one of the words: how, what, why, when, etc. near the beginning of every question. "What is the name we give to the unit of measure that is equal to 1,000 meters?"

(8) **The Yes-No Question:**

Example: Is the radius of a circle twice the diameter?

Criticism: Every pupil has a 50% chance of answering correctly by merely guessing "yes" or "no." The question does not encourage thought, nor does it encourage pupils to use the vocabulary that they have learned.

Suggestion: By what number would you multiply the measure of the radius to find the measure of the diameter?

(9) **The Echo Question:**

Example: There are four quarters in a dollar. How many quarters are there in a dollar?

Criticism: The question calls for only the most rudimentary form of response: parroting. It does not stimulate thought, and a pupil can answer it correctly without really understanding the concept.

Suggestion: Wait for some time to pass before asking pupils to recall a previously developed fact.

(10) **The Question Addressed to a Specific Pupil:**

Example: Robert, can you move the hands on this clock to make it read one-thirty?

Criticism: By starting with the name of a specific pupil, the teacher has turned the discussion into a one-to-one exchange. Other pupils are not encouraged to think about their responses, since the question appears to have been designed for only a particular pupil.

Suggestion: Ask the question, pause to give all pupils time to consider their responses, and then call on a particular pupil. Who can move the hands on this clock to make it read one-thirty? ... Robert?

(11) **The Guessing Question:**

Example: Is 25% equal to one-half or one-quarter?

Criticism: Like the Yes-No question, this type of question limits thinking and creates a situation in which any pupil, regardless of his or her understanding of a concept, has a 50-50

chance of answering correctly.

Suggestion: What common fraction is 25% equal to? OR: Is 25% equal to one-half or one-quarter, and explain why you made your choice.

(12) **The Re-phrased Question:**

Example: What name do we give a half-circle? I mean, when you draw a diameter, it divides the circle into two parts. What do we call each part? When a circle is divided into two halves, what do we call each half?

Criticism: Pupils are still dealing with the first version of the question when they are presented with second and third versions. While listening to these re-phrasings of the original question, thought is diverted and clarity is inhibited.

Suggestion: Ask one good question clearly expressed and remain silent while pupils are considering their responses to it. For example: If a diameter is drawn in a circle, it divides the circle into two equal parts. What names is given to each half-circle?

Pupil Responses

Related to the art of effective questioning is the skill with which the teacher reacts to, or accepts pupil responses. Pupils can be encouraged to formulate their responses in such a way as to make them of value to the entire class. Teachers can further encourage pupil involvement and enthusiasm in the learning process by the ways in which they use pupil responses. Some general guidelines follow:

- Encouraging pupils to comment on each other's responses involves pupils in the learning process to a greater extent than reserving the right to comment as an exclusive right for the teacher: Do you agree with Mario's explanation? How would you correct Mary's answer? What is another way of solving Renee's problem?

- Following up an incorrect response with a sequence of questions designed to help the pupil correct his own answer is much more encouraging than merely skipping over it.

QUESTIONING IN THE MATHEMATICS CLASSROOM

You will find below some questions which might be asked by teachers of grade 3 mathematics. Some of them are well constructed, in good form and in good English. Others are faulty in one way or another. Criticize each question according to the following pattern:

a. If you think the question is a good one, indicate this fact, and explain why you think it is good.

b. (1) If you think the question is a poor one, indicate this fact, and explain why you think it is poor, and

(2) rephrase the question in a way which will make it satisfactory.

1. Mary, what is the solution of ?+8½11?

2. How can we decide if the answer to the subtraction problem is correct?

3. Count backwards by 10, the number following 210 is what?

4. How can we compare a line segment and a ray?

5. Wouldn't you say that the perimeter of a quadrilateral is the sum of the lengths of its sides?

6. How can we decide if regrouping is necessary when adding two numbers?

7. What is the procedure for adding fractions with like denominators?

8. Is (1,4) the same as moving 1 space horizontally and 4 spaces vertically?

9. What is the quotient of $4\overline{)84}$?

10. Which fraction is larger $\frac{1}{4}$ or $\frac{1}{5}$?

11. What about negative numbers?

12. How can we round 251 to the nearest ten?

QUESTIONING IN THE MATHEMATICS CLASSROOM

You will find below some questions which might be asked by teachers of grade 4 mathematics. Some of them are well constructed, in good form and in good English. Others are faulty in one way or another. Criticize each question according to the following pattern:

a. If you think the question is a good one, indicate this fact, and explain why you think it is good.

b. (1) If you think the question is a poor one, indicate this fact, and explain why you think it is poor, and

(2) rephrase the question in a way which will make it satisfactory.

1. Is $\frac{6}{12}$ equivalent to $\frac{1}{2}$?

2. How can we decide the order of $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{5}$?

3. What is meant by an odd number?

4. How do parallel lines and intersecting lines compare?

5. When would the probability of an event be zero?

6. Is 5 a prime number?

7. We can change a fraction with a denominator of 100 to a decimal, how?

8. The difference between a chord and a radius is what?

9. How is a parallelogram different from a rectangle and how is a parallelogram the same as a rectangle?

10. Ricardo, how do we find the perimeter of a rectangle?

11. In what situations are negative and positive numbers used?

12. In how many ways can we arrange 3 different books on a shelf?

QUESTIONING IN THE MATHEMATICS CLASSROOM

You will find below some questions which might be asked by teachers of grade 5 mathematics. Some of them are well constructed, in good form and in good English. Others are faulty in one way or another. Criticize each question according to the following pattern:

a. If you think the question is a good one, indicate this fact, and explain why you think it is good.

b. (1) If you think the question is a poor one, indicate this fact, and explain why you think it is poor, and

(2) rephrase the question in a way which will make it satisfactory.

1. Anthony, how do we round 38,464 to the nearest thousand?

2. How do we find the perimeter of a rectangle?

3. The product of 36 and 48 is what?

4. Is 5 a prime number?

5. What is the difference between congruent figures and similar figures?

6. In what ways can we name an angle?

7. What is the measure of the angle and is it obtuse or acute?

8. How can we find the greatest common factor of two numbers?

9. Why is $\frac{9}{12}$ equivalent to $\frac{3}{4}$?

10. How can we decide if regrouping is necessary when adding numbers?

11. Is division of fractions the same as multiplication of fractions?

12. How can we find the area of a rectangle?

QUESTIONING IN THE MATHEMATICS CLASSROOM

You will find below some questions which might be asked by teachers of grade 6 mathematics. Some of them are well constructed, in good form and in good English. Others are faulty in one way or another. Criticize each question according to the following pattern:

a. If you think the question is a good one, indicate this fact, and explain why you think it is good.

b. (1) If you think the question is a poor one, indicate this fact, and explain why you think it is poor, and

(2) rephrase the question in a way which will make it satisfactory.

1. What is the numerator in $\frac{2}{3}$, and what is the denominator in $\frac{2}{3}$?

2. What is the procedure for rounding decimals in hundredths to the nearest tenth?

3. Is 5 a prime number?

4. How does the procedure for subtracting fractions with unlike denominators different from subtracting fractions with like denominators?

5. James, do you know how to multiply $\frac{2}{3} \times \frac{3}{4}$?

6. What about reciprocals?

7. How does a prime number differ from a composite number?

8. What is meant by a ratio?

9. The sum of the angles of a triangle is what?

10. Is -10 negative or positive?

11. Are the angles of the square each equal to 90&?

12. We compare integers, how?

QUESTIONING--COMMON MISTAKES

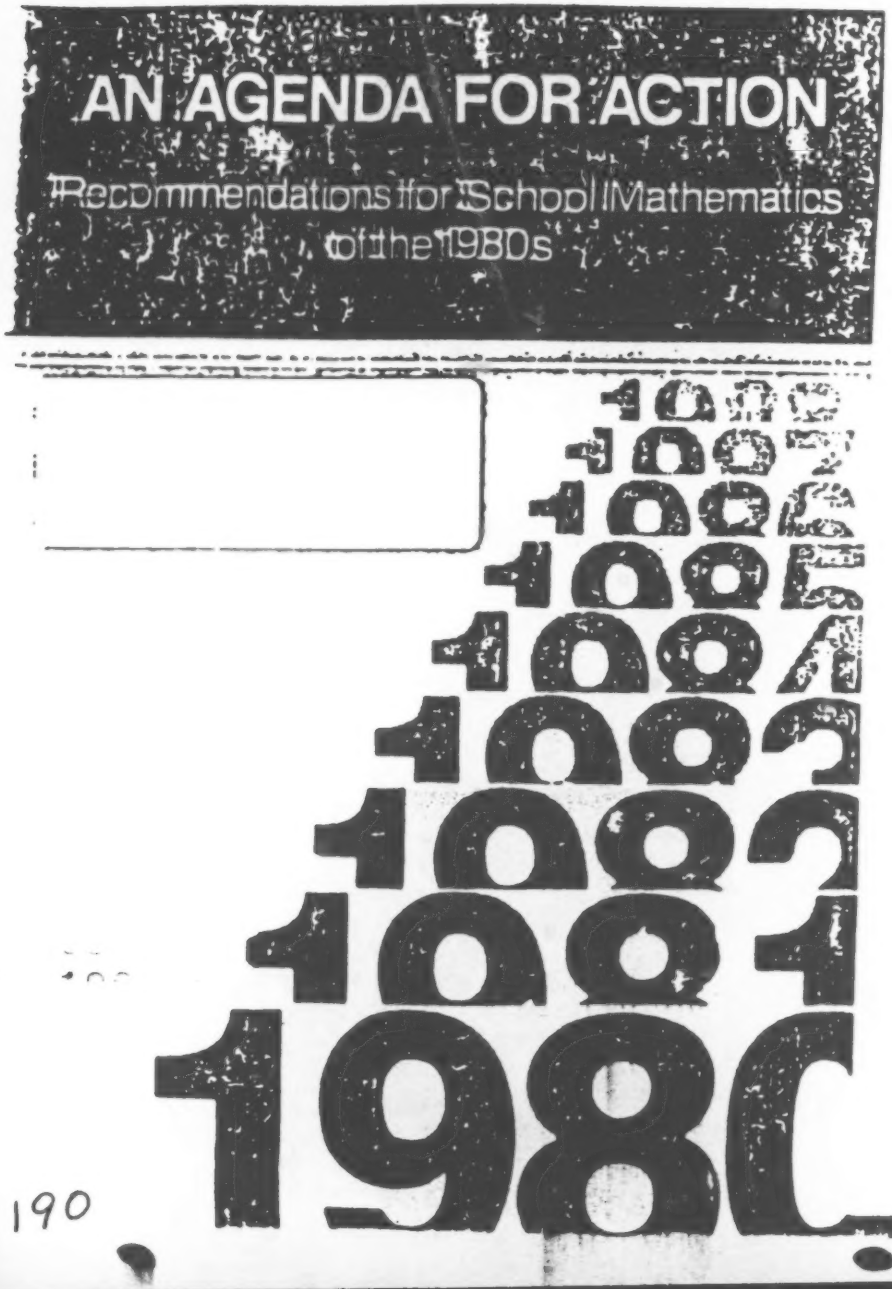
1. The teacher does not distribute the questions evenly among all students.
2. The teacher cannot be heard by all the students.
3. The teacher does not use English correctly when asking questions.
4. The teacher asks irrelevant or meaningless questions.
5. The teacher repeats the answer to the questions he/she asks.
6. The teacher calls on the student first and then proceeds to give the question.
7. The teacher compounds his/her questions, that is, he/she asks two questions successively.
8. The teacher asks a question which commands a Yes or No answer.
9. The teacher asks questions that are too difficult for the students in the class.
10. The teacher does not call on volunteers.
11. The teacher calls on volunteers to the exclusion of non-volunteers.
12. The teacher does not encourage the shy students to answer.
13. The teacher encourages unison answers by using questions that require only a one-word answer.
14. The teacher does not utilize thought provoking questions.
15. The teacher does not correct the incorrect English used by a student in answering a question.
16. The teacher does not grade questions in difficulty to meet the readiness of the class.
17. The teacher does not wait for an answer and give enough time for thought.
18. The teacher does not commend students for correct answers.
19. The teacher does not hold the students responsible for listening to the questions and to the answers.
20. The teacher does not encourage the students to ask questions.
21. The teacher does not use questions to emphasize or summarize a lesson.

Situations for Discussion by New Teachers: Questioning

1. After your first formal observation, your supervisor says that, while you were doing a commendably diligent job, you were monopolizing class discussion and were simply talking too much. What do you do?

2. After you have asked the question, "What is the relationship between the angles of a triangle?" A student who rarely answers vigorously waves his hand and answers, when called upon, "They are congruent." What do you do?

National Council of Teachers of Mathematics



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An Agenda for Action

Recommendations for School Mathematics of the 1980s

The National Council of Teachers of Mathematics recommends that

1. problem solving be the focus of school mathematics in the 1980s;
2. basic skills in mathematics be defined to encompass more than computational facility;
3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;
4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;
5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;
7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;
8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.

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National Council of Supervisors of Mathematics

oblem Solving

Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with non-textbook problems. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. In solving problems, students need to be able to apply the rules of logic necessary to arrive at valid conclusions. They must be able to determine which facts are relevant. They should be unfearful of arriving at tentative conclusions and they must be willing to subject these conclusions to scrutiny.

Applying Mathematics to Everyday Situations

The use of mathematics is interrelated with all computation activities. Students should be encouraged to take everyday situations, translate them into mathematical expressions, solve the mathematics, and interpret the results in light of the initial situation.

Alertness to the Reasonableness of Results

Due to arithmetic errors or other mistakes, results of mathematical work are sometimes wrong. Students should learn to inspect all results and to check for reasonableness in terms of the original problem. With the increase in the use of calculating devices in society, this skill is essential.

Estimation and Approximation

Students should be able to carry out rapid approximate calculations by first rounding off numbers. They should acquire some simple techniques for estimating quantity, length, distance, weight, etc. It is also necessary to decide when a particular result is precise enough for the purpose at hand.

Appropriate Computational Skills

Students should gain facility with addition, subtraction, multiplication, and division with whole numbers and decimals. Today it must be recognized that long, complicated computations will usually be done with a calculator. Knowledge of single-digit number facts is essential and mental arithmetic is a valuable skill. Moreover, there are everyday situations which demand recognition of, and simple computation with, common fractions.

Because consumers continually deal with many situations that involve percentage, the ability to recognize and use percents should be developed and maintained.

Geometry

Students should learn the geometric concepts they will need to function effectively in the 3-dimensional world. They should have knowledge of concepts such as point, line, plane, parallel, and perpendicular. They should know basic properties of simple geometric figures, particularly those properties which relate to measurement and problem-solving skills. They also must be able to recognize similarities and differences among objects.

Measurement

As a minimum skill, students should be able to measure distance, weight, time, capacity, and temperature. Measurement of angles and calculations of simple areas and volumes are also essential. Students should be able to perform measurement in both metric and customary systems using the appropriate tools.

Reading, Interpreting, and Constructing Tables, Charts, and Graphs

Students should know how to read and draw conclusions from simple tables, maps, charts, and graphs. They should be able to condense numerical information into more manageable or meaningful terms by setting up simple tables, charts, and graphs.

Using Mathematics to Predict

Students should learn how elementary notions of probability are used to determine the likelihood of future events. They should learn to identify situations where immediate past experience does not affect the likelihood of future events. They should become familiar with how mathematics is used to help make predictions such as election forecasts.

Computer Literacy

It is important for all citizens to understand what computers can and cannot do. Students should be aware of the many uses of computers in society, such as their use in teaching/learning, financial transactions, and information storage and retrieval. The "mystique" surrounding computers is disturbing and can put persons with no understanding of computers at a disadvantage. The increasing use of computers by government, industry, and business demands an awareness of computer uses and limitations.

HOW TO SOLVE IT

First.

You have to *understand* the problem.

UNDERSTANDING THE PROBLEM

- *What is the unknown? What are the data? What is the condition?*
- Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?
- Draw a figure. Introduce suitable notation.
- Separate the various parts of the condition. Can you write them down?

Second.

Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.

DEVISING A PLAN

- Have you seen it before? Or have you seen the same problem in a slightly different form?
- *Do you know a related problem?* Do you know a theorem that could be useful?
- *Look at the unknown!* And try to think of a familiar problem having the same or a similar unknown.
- *Here is a problem related to yours and solved before. Could you use it?* Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?
- Could you restate the problem? Could you restate it still differently? Go back to definitions.
- If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other?
- Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

Third.

Carry out your plan.

CARRYING OUT THE PLAN

- Carrying out your plan of the solution *check each step.* Can you see clearly that the step is correct? Can you prove that it is correct?

Fourth.

Examine the solution obtained.

LOOKING BACK

- Can you *check the result?* Can you check the argument?
- Can you derive the result differently? Can you see it at a glance?
- Can you use the result, or the method, for some other problem?

PROBLEM SOLVING STRATEGIES

The emphasis in the strategies that follow is not on how to solve problems, but on how to read and study problems so that they may be solved. Students should be given the opportunity to try several sets of strategies. Note problem solvers get stuck because they have fixed on a single procedure or approach.

The following are guidelines for teachers in helping students solve problems:

1. Make sure students understand the problem.
   a. Do the students understand the meanings of the terms in the problem?
   b. Do the students take into consideration all the relevant information?
   c. Can the students indicate what the problem is asking them to find?
   d. Can the students state the problem in their own words? If appropriate, can the students explain the problem in terms of a sketch?
2. Help students gather relevant thought material to assist in creating a plan.
   a. Assist the students in gathering information by having them analyze the given conditions (and sometimes the assumed solution).
   b. Help students obtain information by analyzing an analogous problem.
   c. When students have become discouraged by pursuing an unproductive approach, help them view the problem from a different perspective.
3. Provide students with an atmosphere conducive to solving a problem.
4. Once students have obtained a solution, encourage them to reflect on the problem and the means of solution.
   a. If possible have students verify solutions that have not been established deductively.
   b. Encourage students to seek and present alternate ways of solving a problem.
   c. Challenge students to investigate variations of the given problem.

CHECKLIST FOR CREATING A PROBLEM SOLVING ATMOSPHERE

The purpose of the following survey is to examine your classroom teaching to determine whether the learning of problem solving is taking place. If you will honestly and thoughtfully mark all of the statements that follow, you will be able to be more aware of a classroom atmosphere that includes a emphasis on problem solving.

DO YOU		YES	NO
1.	create an impression that the teacher is also investigating the problem?	____	____
2.	foster an exchange of ideas among students.	____	____
3.	ask a sequence of questions which lead students to discover a concept?	____	____
4.	encourage students to explore original ways of solving problems?	____	____
5.	make clear that a variety of ways exist to solve problems by asking, "Can we think of another way of doing this?"	____	____
6.	focus students' attention on process of thinking?	____	____
7.	encourage students to make up problems?	____	____
8.	promote divergent thinking?	____	____
9.	avoid lecturing?	____	____
10.	avoid asking, "Are there any questions?" or "Is there anyone who doesn't understand?"	____	____
11.	avoid allowing students to shout out answers spontaneously?	____	____
12.	avoid using responses of more able students to dictate pace of class?	____	____
13.	ask class to react to an answer given by a student rather than teacher indicating approval or disapproval?	____	____
14.	listen to students?	____	____
15.	encourage students to verbalize their thinking?	____	____
16.	encourage students to defend or justify answers?	____	____
17.	encourage students to estimate and determine reasonableness of answers?	____	____

18. show more interest in the process of solving a problem rather than the answer? ____ ____

19. motivate students to want to think? ____ ____

20. give students time to think? ____ ____

21. offer activities which permit students to explore skills and/or concepts before trying to find a solution? ____ ____

INCORPORATING PROBLEM SOLVING SKILLS IN YOUR CLASSROOM

The more opportunities that students have for solving problems, the more skillful they will became as problem solvers. Time should be allotted during each class lesson to the skills of problem solving as well as to their application. Some suggestions are:

- Have students prepare their own original problems from a given set of data.
- Start each lesson with a problem solving experience.
- Be sure that each homework assignment contains at least one problem solving activity.
- Start a class book of problems. Bring in charts, menus, price lists, pictures, listings of information, graphs and tables gathered from newspapers and magazines. Ask pupils to write and solve problems related to these materials.
- Classroom activities such as collecting or spending money, team scores, cake sales, health data, planning for a trip or party opportunities for generating and solving verbal problems.
- Use student prepared problems for homework. Identify the problems with the student's name.
- Have students who wrote the problem check the answers submitted.

EFFECTIVE TEACHING OF PROBLEM SOLVING

When visiting a classroom in which the learning of problem solving is taking place, one would expect the teacher to:

1. have carefully planned the lesson.
2. encourage students to verbalize their thinking.
3. ask a sequence of questions which lead students to discover a concept.
4. foster an exchange of ideas among students.
5. create the impression that the teacher is also investigating the problem.
6. encourage students to explore original ways of solving problems.
7. make clear that a variety of ways exist to solve problems by asking, "Can we think of another way of doing this?"
8. focus students' attention on the process of thinking.
9. promote divergent thinking.
10. avoid lecturing.
11. avoid asking "Are there any questions?" or "Is there anyone who doesn't understand?"
12. avoid allowing students to shout out answers spontaneously.
13. avoid using responses of more able students to dictate pace of class.
14. ask class to react to an answer given by a student rather than teacher indicating approval or disapproval.
15. encourage students to defend or justify answers.
16. encourage students to estimate and determine reasonableness of answers.
17. show more interest in the process of solving a problem rather than the answer.
18. motivate students to want to think.
19. encourage students to ask questions.
20. give students time to think.
21. listen to students.
22. offer activities which permit students to explore skills and/or concepts before trying to find a solution.

STRATEGIES FOR IMPROVING THE CLASSROOM CLIMATE FOR PROBLEM SOLVING

1. Emphasize ideas such as
   - problem solving takes time and patience
   - it is OK to make mistakes; sometimes errors help one arrive at a solution
   - a student who makes progress toward a solution has been successful
   - it's really fun and satisfying when you try hard and solve a problem
2. Give a story or a problem and have students act out the situation and phrase the question in their own words. Use objects when helpful.
3. Give students a number sentence and encourage them to formulate a problem that can be solved using the sentence. Post student's problems on a bulletin board for other students to solve, or make a weekly newsletter containing problems that students have created for others to solve.
4. Ask questions to help students understand the problem better.
5. Ask questions that give the students hints for solving when needed.
6. Ask questions that encourage the students to describe and analyze their solutions.
7. Compliment students for taking time to think through a problem rather than for how quickly they find a solution.
8. Use detective or mystery stories as an aid to help students problem-solving skills.
9. Look for everyday situations about which you can pose questions that stimulate thinking. e.g. At about what time will the hands on a clock be at right angles to each other?
10. Have students explain their plan and solution.
11. Have students look for and discuss alternate solutions to the same problem.
12. Help students build problem-solving skills by making new problems from a given problem. e.g. Change one or more of the conditions of the problem; interchange the "given" and the "to find".
13. Personalize problems. Use the names of your students and of local places to make problems come alive.
14. Encourage students to choose things they are interested in and write problems about them for other students to solve.
15. Have students work in groups and solve problems so that they can discuss the strategies possible to solve the problems.
16. Do not reject guesses;pursue them.

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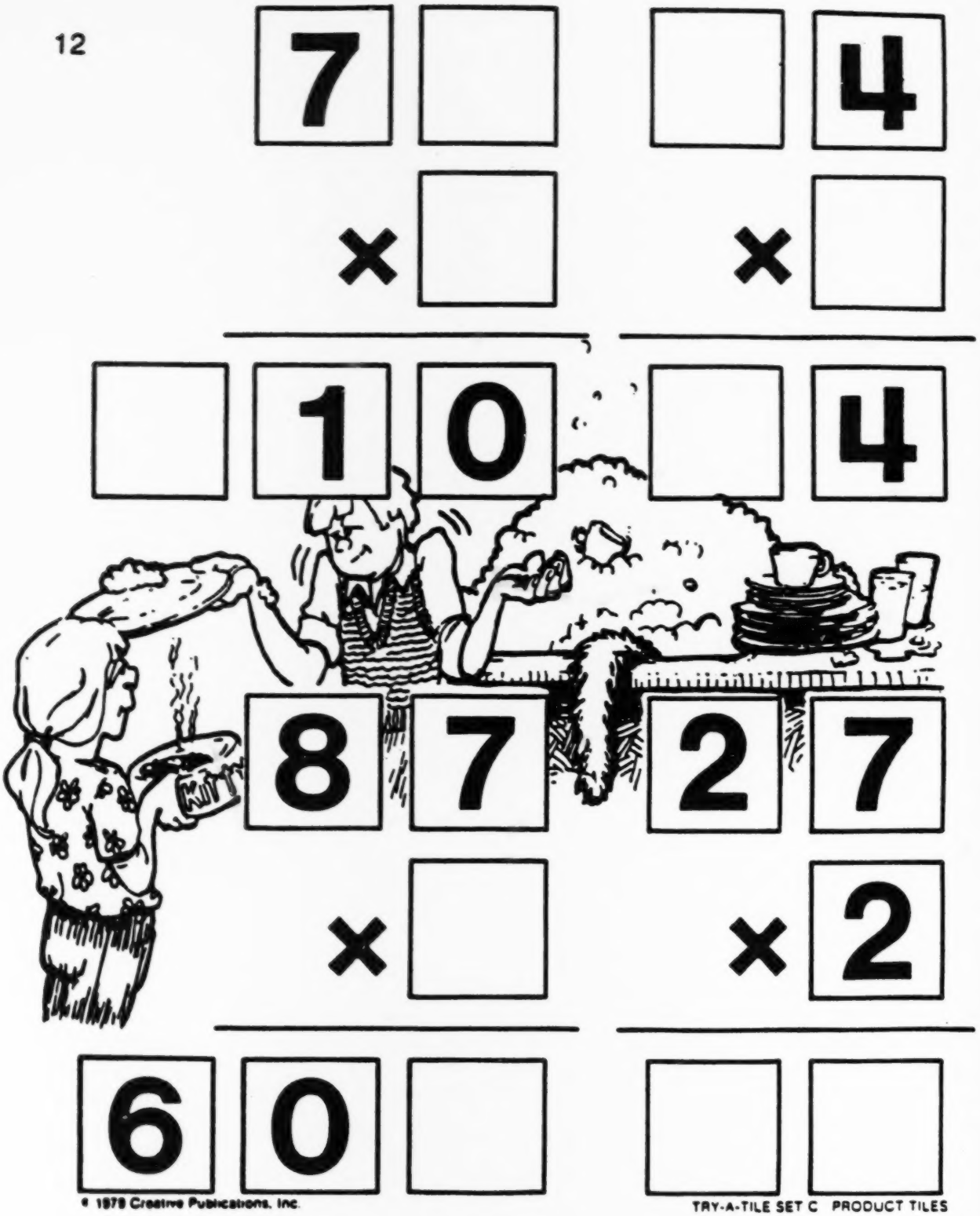

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TRY-A-TILE SET C PRODUCT TILES

HOW MANY WAYS CAN YOU MAKE CHANGE FOR ME WITHOUT USING PENNIES?

	5¢	10¢	25¢	50¢
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
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HOW MANY WAYS CAN YOU CHANGE A $50 BILL USING 5, 10, AND 20 DOLLAR BILLS?

Some Additional Problem Solving Strategies

Analyze Language: Problem solving often becomes an exercise in vocabulary. There are three classes of words or phrases which the teacher must be alert for:

a) words which come from mathematics only and have only technical meanings, such as sum, quotient, numerator, etc.,

b) words which have both technical meanings and broader meanings in everyday usage, such as product, square, area, interest, etc.,

c) key words and phrase which translate into mathematical concepts and notation, such as less than, greater than, equal to, increased by, decreased by, difference between, etc.

Efforts to lead students to understand the vocabulary of verbally stated problems will open up avenues of approach in solving them. Special vocabulary items of the kinds mentioned above should be listed with their meanings and reviewed and reinforced frequently. A special section of students' notebooks can be devoted to keeping an on-going glossary of terms encountered in verbal problems. Vocabulary items can be included in homework assignments and tests.

- Guess and Check: What number when multiplied by itself or when added to zero gives the same result? Have students guess numbers that seem to be possible answers and test them to see if they satisfy the conditions of the problem? Does 2 x 2 equai 2 + 0?

  Does the number 5 work? Does 1 work? Does 0 work? Students may discover that there is more than one such number. Or they may discover, in other problems, that there is no such number.

- Draw From Students' Background: "Mrs. Rodriguez wanted to buy a poound of potatoes costing $1.15, 2 pounds of steak at $3.50 a pound, and a cake for $1.50. Why did the store owner tell her that her $10 bill was not enough?" Lead students to recall actual visits to the supermarket. Ask if they ever had the experience of having to pay more for an item than the price that was marked on it.

- Make a Simpler Problem: "What is the largest number of rectangles that can be created by adding four line segments to a rectangle?" After agreeing that this is quite an involved question, have the class consider a more elmentary form of the same idea: "What is the largest number of rectangles that can be created by adding one line segment to a rectangle?" Then the class might consider two line segments, three and, finally four. Perhaps, before four line segments are finally considered, someone will have found a pattern and will be able to state a generalization for any number of additional line segments.

- Work Backwards: "Martin, Sandy and Alice each have some money. Martin gives Sandy two dollars and Alice one dollar. Then Sandy gives Martin three dollars and Alice one dollar. Finally, Alice gives Sandy two dollars and Martin one dollar. At the end of this process, Martin, Sandy and Alice each have twelve dollars. How much money did each of them start with?" This problem illustrates how organized thinking can lead smoothly to the solution of a problem which, at first, seems hopelessly confusing. In this case, the final outcome is known: each person ends up with twelve dollars. Beginning with this, it is possible to work backwards, step-by-step, to trace the amounts of money each person has at each of the four stages of the money exchange. The movement of dollars can be traced by working with a table such as the following, beginning at the bottom and working upward:

Martin	Sandy	Alice	
$12	$12	$12	(final situation)
$11	$10	$15	
$8	$14	$14	
$11	$12	$13	(amount of moeny each started with)

Notice that the total amount of money is $36. While the dollars change hands, the total number of dollars doesn't change.

PROBLEM SOLVING

In 1-12, tell whether you would use addition, subtraction, multiplication, or division to solve the problem in the shortest way.

1. If you know the cost of one shirt, find the cost of several such shirts.

2. If you know the depths of two lakes, find out how much deeper one lake is than the other.

3. A man travels the same number of miles to and from work on each working day. If you know this number of miles, find how far he travels on these trips during a 5-day work week.

4. If you know how much you paid for a given number of pounds of apples, find the cost of one pound of these apples.

5. If you know the length of a submarine and how much longer a cruiser is, find the length of the cruiser.

6. If you know the profits of a company that are to be shared equally among three partners, find the amount each

7. If you know the number of pairs of stockings in a box, find the number of pairs of stockings in a given number of such boxes.

8. If you know the number of pupils in a school and how many of these are boys, find the number of girls.

9. If you know how fast a car traveled and how much faster a plan flew, how fast did the plane fly?

10. If you know the rate of speed at which a ship is sailing and the number of hours it sails at this rate, find the distance it travels.

11. If you know how many copies of a newspaper are needed and how many copies are printed in one hour, find the number of hours needed to print the required number of copies.

12. If you know how much you agreed to pay for an article and how much you gave as a deposit, find the amount partner is to receive.you still have to pay.

PROBLEM SOLVING

Tell whether you would add, subtract, multiply, or divide to solve the problem.

1. Ruby, Britt, Lisa, Sean, Joan and Bill shared the expenses of a party. The party cost $24.72. How much was each one's share?

2. Jim bought a steak for $3.59, a bag of potatoes for $.79 and a loaf of bread for $.43. How much did he spend?

3. Joe bought a sweater for $12.99. He gave the cashier a $20 bill and 4 pennies. He received $7.05 change. Did Joe get the right change?

4. Mrs. Ling bought a 1-Kilogram can of coffee at a 25¢ off sale. The regular price was $6.14. How much did she pay?

5. Maria bought 5 kilograms of butter to make cookies. Butter was selling for $1.48 per kilogram. How much did Maria pay?

6. Apples were selling at 3 kilograms for $1.00. Mr. O'Neill bought 1 kilogram of apples. How much did he pay?

PROBLEM SOLVING

What else do you need to know before you can solve the problem?

1. A dozen golf balls are maked down $2.50. What is the cost per dozen?

2. A youth group washed 23 cars to earn money. How much did they earn?

3. Dog food is sold in 6-can packages. How much would 15 cans cost?

4. Wool material was on sale for $4.25 a meter. How much did it cost Dee to make a skirt?

5. A scout group needs $16.00 to go on a camping trip. How much will each scout have to give?

6. Doug's plane ticket cost $97. How much per kilometer did his trip cost?

7. Ms. Gomez drives 60 kilometers from her business to her vacation home. How long does it take?

8. Betty bought eggs for 87¢ and a bag of potatoes for $1.39. How much change did she get?

PROBLEM SOLVING

In 1-10, a fact needed to solve the problem has been omitted. State what this fact is.

1. Ruth paid 55 cents for a box of sugar. What was the cost of a pound of sugar?

_____

_____

2. If you bought two ties, how much did you pay for them?

_____

_____

3. Mr. Roberts saves the same amount of money each week in order to buy a television set. How long will it take him to save enough money to buy a television set that costs $190?

_____

_____

4. Sue bought a bag of potatoes for 60 cents. How much did she pay for a pound of potatoes?

_____

_____

5. If you paid $90 for 1 coat and 1 dress, what did you pay for the coat?

_____

_____

6. At 69 cents per gallon, what is the cost of a tankful of gasoline?

_____

_____

7. Harry took a trip in which his car averaged 15 miles per gallon of gas. How many gallons of gas did he use on the trip?

_____

_____

8. Mr. Wise bought an FM radio costing $120. He made a down payment of $30, and paid the rest in equal weekly payments. How many weekly payments did he have to make?

_____

_____

9. Mr. Breen spent 8 hours installing storm windows. On the average, how long did it take him to install one storm window?

_____

_____

10. Marian bought 6 pairs of stockings. How much change should she receive from a $20 bill?

_____

_____

PROBLEM SOLVING

What question could be asked about this information?

1. Mr. Overdraw wrote checks for these amounts:
   $192.50, $64.50, $18.53, $80.00, $13.64, and $83.50.

   Example answers: How much did Mr. Overdraw pay out?

   or

   How much more than $400 did he pay out?

   or

   What is the difference between the largest amount and the smallest amount?

2. The "Know It All Newspaper" has a daily circulation of 250,000 papers and a Sunday circulation of 325,000 papers.

3. Ms. Gray Eagle sold some clothing for $925. The clothing cost $560. Ms. Gray Eagle's overhead was $305.

4. The distance from the fulcrum to one end of a lever is 1 meter. The distance from the fulcrum to the other end of the lever is 15 centimeters.

5. Dot and Joe left Little Falls at the same time. Dot traveled north and averaged 72 kilometers per hour. Joe traveled south and averaged 64 kilometers per hour.

6. Mr. Kanelos paid for these items with a $10 bill.

Meat	$3.15
beans	.50
sugar	.42
cereal	.60
bread	.66

7. A mail carrier stops at 15 houses on a block. She walks 4 meters from the corner to the mailbox of the first house. She then walks 18 meters between the mailboxes of each house thereafter.

PROBLEM SOLVING

What information is not needed to solve the problem?

1. A bag of 5 grapefruit sold for 89¢. A bag of 6 oranges sold for 79¢. How much did Willie pay for 3 bags of oranges?

2. How much must Nadine pay for 3 pounds of grapes which come in 25-pound boxes and cost 49 cents per pound?

3. A car which weighs 3630 pounds travels at an average rate of 50 miles per hour. How far does it go in 4 hours?

4. How many shelves each 3 feet long can be cut from a 10-foot piece of lumber which costs $4?

5. Find the cost of a set of 12 napkins, each 16 inches by 16 inches, if the cost of each napkin is $3.

6. A salesman receives a weekly salary of $100. In addition he receives $3 extra for each article he sells. One week he sold 24 articles. How much extra did he receive that week?

7. Mrs. Basso's electric bill was $31.45. Her heating bill was $63.78, and her phone bill was $18.27. How much did Mrs. Basso spend on heat and electricity?

8. Carl left his home at 8:00 A.M. He averages 30 kilometers per hour and arrived at the ski lodge at 1:00 P.M. How long did the the trip take?

9. Rose left her house at 9:00 A.M. She averaged 25 kilometers per hour and drove for 8 hours. How far did she drive?

10. Amir is 1.6 meters tall and weighs 60.7 kilograms. Bill is 1.5 meters tall and weighs 80.6 kilograms. How much more does Bill weigh than Amir?

11. Debbie weighs 54.3 kilograms. She bought 5 kilograms of potatoes, 1.5 kilograms of meat, and .5 of a kilogram of butter. What was the total weight of the potatoes, meat, and butter?

12. Mr. Wolf's rent is $250 per month. His food bill averages $135 per month, and his commuting expenses amount to $30 per month. How much are his monthly rent and food expenses?

PROBLEM SOLVING

Enough, too much, or not enough information?

1. Who had the higher bowling score?

Game	Robert	Sidney
1	97	108
2	132	128
3	148	156

2. How much profit did the Bad Food Diner make?

Day	Receipts
Monday	$1,611.28
Tuesday	$1,247.55
Wednesday	$ 982.00

3. A piece of drill rod is 46 centimeters long. How many pins can be cut from it?

4. Ms. Regis used 200 liters of gasoline in July. She charged these amounts: $8.75, $10.50, $8.42, $3.58 and $7.24. What was the total cost of the gasoline?

5. Earth's diameter measures about twice as long as Pluto's diameter. What is the measure of Pluto's diameter?

6. Mrs. Brown died after paying 13 premiums of $65 each on an $8,000 life insurance policy. Mr. Brown received $8,000. The Browns had 3 children. How much more did he receive than the amount paid out?

7. Dora averages 19.2 kilometers on 3.8 liters of gasoline. How far can she travel on a full tank of gas?

8. Alex traveled 144 kilometers and used 19 liters of gasoline. How many kilometers per liter did he average?

HOMEWORK

Questions to consider:

a. How long?

b. Should it be cyclical?

c. Should it be graded in difficulty?

d. Should it be adapted for the level of the class?

e. When should it be prepared?

f. How is it given to the student?

g. What if there no text/poor texts?

h. Is it reviewed in class?

i. Are all problems reviewed?

j. How much time is spent in class reviewing the homework?

k. Who writes the problems on the board?

l. Who corrects the wrong solution?

m. Is it collected?

n. Is it returned?

o. What part of the grade does homework reflect?

p. Is parent notified of missing homework?

q. What about students who never do the homework?

ON HOMEWORK

HYPOTHESIS: Homework which reaches optimum significance needs to be carefully planned to harmonize with instructional goals. It should accent quality more than quantity. It should prevent wasted or uneconomical effort by students. It should be cyclical in nature to assure retention of previously learned material.

CHARACTERISTICS OF A POOR ASSIGNMENT:

1. The assignment consists of an arbitrary designation of pages or exercises in the text, chosen hurriedly and with little thought.

2. Prerequisite knowledge and skills have not been identified. There is no reason to believe the students can handle the material.

3. The assignment is fragmentary and bears little relation to what has gone before or what is to come after.

4. Although drill is not the goal, there is repetition to the point of frustration or boredom. No challenging exercises are present.

5. The assignment has not been planned for utilization in subsequent lessons; it doesn't set the stage for new ideas.

PLANNING VERSUS MAKING A HOMEWORK ASSIGNMENT:

Teachers should make a conscious effort in their planning:

a. to investigate future topics
b. to reinforce previous topics
c. to provide experiences that promotes discrimination among concepts in order to make the mathematics more available for the solution of the problems arising outside the mathematics classroom.

STEPS TO CONSIDER IN PREPARING CYCLICAL HOMEWORK ASSIGNMENTS:

1. The teacher should first list the topics that are to be taught each day (for the days between full-period tests).

2. The homework assignments are then planned <u>vertically</u> in the following manner: <u>ONLY TOPICS ARE LISTED FIRST</u>.
   a. the topic on the day's lesson is included for at least 3 successive days and then including it every other night or every third night, etc.
   b. The other topics chosen are from previous lessons alternating when they are given. Continuous computational topics should be included in most classes.

   c. After the topics have been listed for each assignment, the teacher should consider the length of the assignment and whether enough topics are covered for the needed variety on the test following these set of assignments. If adjustments are necessary, additional review from previous work can be included. Also movement of topics (because of too many long questions in one assignment versus not any long question, etc.) may be done at this point. Teachers are reminded that homework must be long enough to cover review but short enough to have students explain their solutions when it is gone over and yet allowing time for the teaching of a new developmental lesson each day.

d. After this planning, the books can now be used to select the problems. If this is done in a vertical fashion considering one topic and selecting problems for all of the assignments on this topic, gradation and variety on successive nights is made easier.

Each assignment should include at least 1 question that is verbal in nature.

Each assignment should contain in both single concept questions as well as multiple concept questions.

The new material should be lightly stressed in the homework. Giving 1 problem on the new topic is more successful than giving many. In this way, greater stress can be placed upon asking students to verbalize the rules, methods, or procedures needed to solve such a problem. If a student cannot do this problem, giving more than one problem will cause further frustration and math anxiety. If a student makes an error, he would reinforce this error many times if more than one problem is given. Giving one problem also provides a quick way of determining whether the new material was learned. Variations of the topic are given in successive nights.

e. Each assignment includes pages to be read on the topic taught that particular day.

f. Teachers should realize that if there is any uncertainty of completing a topic on a particular day, it is better not to include this topic on that night's assignment.

ADDITIONAL COMMENTS WITH REFERENCE TO HOMEWORK:

1. If homework assignments are planned from test to test,

   a. teachers will not only know whether they have enough topics for a varied and comprehensive test but will also know if they have both long and short topics for the given test.

   b. teachers can include review of topics previously tested that in their judgment need further review spirally in future homeworks and then include these topics on the next test. This should help the student who was unsuccessful on a given topic when it was first tested become confident when they are successful on the next test.

   c. students become more familiar with a comprehensive test and not as thrown when they take a state exam or a city-wide exam.

   d. the teacher can make up the test at the same time that they have prepared the set of homeworks between tests. It this way the test can be written, duplicated, proofread and a key prepared well ahead of it being given. Teachers are then directed as to what schedule of teaching should occur and in some way prevents going off on a tangent in class.

2. The overhead projector is very helpful in going over the homework. Students may purchase china markers at the beginning of the term. The teacher then gives an acetate when assigning problems to each student to bring in the following day. The student then comes to the machine, explains his/her work, chairing any questions asked from the class. This furthers verbalization by students and provides more sustained explanations. Teachers need to stress that students give explanations of what was one or thought of to go from step to step rather than have them read the work. No time is spent putting up the problems

on the board. All work becomes visable to the entire class. More student interaction takes place since this student is chairing questions from the class. Students may borrow the acetate and bring in the next day. If a disruption during a period (fire drill, etc.) the homework may be gone over the following day. Time may be spent on training students to pay attention to the discussion on hand not to copying of the work. More emphasis can be placed on re-doing work done incorrectly rather than mere copying of the solution. Going over the homework after the applications of the new lesson and before the summary of the lesson usually speeds up the going over of the homework rather than extending if to the entire period or most of the period.

3. Teachers are asked to prepare all solutions to each assignment in advance of their need with the students. In this way the teacher can determine the types of errors that could occur and know what to stress in the teaching of the topic. Time is saved since the teacher can quickly determine the point of error and ask questions accordingly.

4. Teachers may prepare homework sheets for distribution to the class. The benefits of having such a sheet are as follows:

   a. for students:
      (1) if absent, they have the homework and reading assignment and can come in prepared
      (2) provides information as to what topics to study for a test
      (3) gives enough advanced notice of test date to allow for planned study periods
      (4) if they know of another commitment for a particular night, they can do their homework in advance

   b. for teachers:
      (1) eliminates need for writing assignment on the board at the beginning of each class period
      (2) eliminates nightly pressure
      (3) eliminates having to first review which problems were already given
      (4) allows for gradation of problems
      (5) allows for a better planned test
      (6) provides material to be given to new admits and those students who return after extended absence.

   c. for parents:
      (1) gets better image of school, department and teacher since they see more planning
      (2) appreciates that child can do work if absent
      (3) appreciates not having last minute notice of test

Sample model of information on homework sheet:

#23 Given: Due:

Name of Text	104	201	204	212	277	313	Page #
Read 236-8,249, 250,305,306	46	15	48	13	19	16	Problem #

Answers to problems:

Note:

Putting answers on sheet helps student learn how to use answers when doing solutions and reduces math anxiety.

THE FOLLOWING MAY BE DISCUSSED WITH STUDENTS AND GIVEN TO THEM IN PRINTED FORM FOR REFERRAL AT HOME WHEN DOING THEIR MATHEMATICS HOMEWORK:

HOW TO DO YOUR MATHEMATICS HOMEWORK

PURPOSES OF HOMEWORK:

a. To help reinforce what you learned in class.
b. To help identify those parts of the lesson you do not understand.
c. To increase the number of correctly solved problems to which you can refer when review ing for a test.

1. Read the introductory section of the assignment to identify and understand the principle it teaches.

   a. Make certain you understand the meaning of every word in the section.
   b. Visualize every step along the way.

2. Do the sample exercises.
   This will determine how well you understand the principles being taught and whether you will be able to work problems based on those principles.

   a. Read each step carefully in the model problems.
   b. Understand every term used.
   c. Follow the reasoning behind each manipulation.
   d. Solve the problem all over again on your own.

   It is a good idea to reserve a section of your loose-leaf notebook for working through the sample problems in the book.
   Writing out the steps is far better than merely reading through what the author has written.
   This section in your notebook will be a handy review for you before tests.

3. Recite the principles you have learned. If you cannot repeat the principles used, go back to the reading material prior to the exercises and read it through again.
   Write the principles and procedures used to solve each type of problem on an index card from which you can study for a test as well as quickly review previous work each day.
   Make review a part of your daily calendar. It will eliminate last minute cramming and further retention of your mathematical skills.

What to do if you "get stuck" on a problem!

a. Look back at the book, at your notes, and at the sample model problems.

What to do when an answer is wrong in your homework.

a. Analyze the problem to find out at what point the error was made.
b. Listen to the analysis done in class.
c. Take down the correct answer.
d. At home, re-do the problems you got wrong before trying the new assignment.

GRADE 3 HOMEWORK ASSIGNMENT PLANNING

Homework Planning By Topic

Date	Topics of Lessons to be Taught	Current Topics					
	Unit 40 Addition fractions (like den.) Subtraction fractions (like denom)	Add.like fractions					
	Unit 40 Solving verbal problems involving + and - like fractions	Subt.like fractions					
	Unit 41 + and - decimals to tenths (no regrouping)	Add.like fractions (verbal)	+ dec.to tenths (no regrping)				
	Unit 41 Solving verbal problems involving + and - decimals to tenths	Subt.like fractions (verbal)	- dec.to tenths (no regrping)				
	Unit 42 Predict the outcomes of an event		+dec.to tenths (no regrping)	Predicting outcomes			
	Unit 42 Predict the outcomes of unequally likely events	Add.like fractions		Predicting Outcomes			
	Unit 43 Mult. of sixes		- dec.to tenths (no regrping) (verbal)	Predicting outcomes	x sixes		
	Unit 43 Mult. of sevens Commutative property involving 6 & 7 times tables	Subt.like fractions			x sixes	x sevens	
	Unit 43 Solving word problems involving mult. of sixes and sevens		+ dec.to tenths (no regrping) (verbal)	Predicting Outcomes	x sixes (verbal)	x sevens	
	Unit 44 Division by 6 (dividends to 54)	Add.like fractions (verbal)	- dec.to tenths (no regrping)		x sixes	x sevens (verbal)	$6\frac{1}{8} \div 6$
	Unit 44 Division by 7 (dividends to 63)	STUDY FOR TEST					

non-unit fract.parts of a group (verbal)		÷ by 3 thru 27	x 2		3 digit - 3 digit (regrp.hund. to tens)	3 digit + 2 digit (regrp.ten to hund.
	order like fract.	÷ by 2 thru 18	x 3	3 digit +3 digit (2 regrp)		
	order unlike unit fract.		x 4			
		÷ by 4 thru 36	x 5		3 digit -3 digit (regrp.hund. to tens)	
non-unit fract.parts of a group (verbal)		÷ by 5 thru 45	x 3 (verbal)			3 digit +3 digit (regrp. tens to h
	order like fract.	÷ by 2 thru 18	x 2	3 digit +3 digit (2 regrp) (verbal)		
		÷ by 3 thru 27	x 5		3 digit -3 digit (regrp.hund. to tens-verbal)	
	order unlike unit fract.		x 4 (verbal)			
non-unit fract.parts of a group (verbal)		÷ by 5 thru 45	x 3	3 digit +3 digit (2 regrp)		
y		÷ by 4 thru 36	x 2		3 digit -3 digit (regrping hund.to tens-verbal)	

GRADE 3 POSSIBLE TEST TOPICS CORRELATED TO HOMEWORK PLANNING SHEET

Non-unit fractional parts of a group(verbal and computation)

'ddition or Subtraction like fractions(computation & verbal)

Ordering like fractions

Ordering unlike unit fractions

Addition or subtraction decimals to tenths--no regrouping(computation & verbal)

Predicting outcomes

Multiplication by 2, 3,4, 5,6, & 7(computation & verbal)

Division by 2, 3, 4,5, & 6

Addition 3-digit and 2-digit--regrouping tens to hundreds (computation and verbal)

Addition 3-digit and 3-digit--1 regrouping and 2 regroupings (computation and verbal)

Subtraction 3 digit from 3- digit--regrouping hundreds to tens (computation and verbal)

GRADE 4

HOMEWORK ASSIGNMENT PLANNING

Homework Planning By Topic

Date	Topics of Lessons to be Taught	Current Topics
	Integers-real life situations	Integers real life situations
	Integers-- word problems	Integers - word problems
	Arrangements of a group of objects	arrangements
	Problem solving-locating patterns on a chart/table	integers-real life situations; locating patterns on a chart/table
	Common factor of 2 or more whole numbers, greatest common factor of 2 or more whole nos.	arrangements; common factor
	Common multiples of a set of whole nos. Least common multiple of a set of whole nos.	integers-word problems; locating patterns on a chart/table; GCF; common mult.
	Area rectangle	arrangements; common factor; LCM; Area rect.
	Volume of rect. prism	locating patterns on chart/table; GCF; common mult.; Area rect.; Vol. rect. prism
	Add money values up to $9.99, Addition of Decimals thru hundredths	integers-real life situations; arrangements; common factor; LCM; Vol. rect. prism
	Addition of Decimals-thru hundredths (word problems)	integers-word problems; locating patterns on chart/table; GCF; common mult.; Area rect.
	Subtract money values up to $9.99, Subtract decimals thru hundredths	STUDY FOR TEST

Review Topics

+ 4 digit	$\frac{?}{100}$ to dec.	÷ 1 digit no remainder	Parallel, intersect., Perpend.Lines	improper fract
Place value of 5 digit nos.	order unit fract.			Per
- 5 digit without regrping	$\frac{.?}{100}$ to dec.		prime no.	
		÷ 1 digit no remainder		mixed no.
+ 4 digit			Parallel, intersect., Perpend. lines	Per.
	order unit fract.		composite no.	
- 5 digit with regrp.				improper to mixed
		÷1 digit no remainder		Per
+ money to $9.99			Parallel, intersect., perpend.lines	improper to mixed
+ dec thru hundredths	order unit fractions.		prime no.	

GRADE 4 POSSIBLE TEST TOPICS CORRELATED TO HOMEWORK PLANNING SHEET

Integers--real life situations & word problems

Arrangements of a group of objects

Locating patterns in a chart/table

Common factor and greatest common factor

Common multiple and Least common multiple

Area rectangle

Volume of rectangular prism

Addition of moeny values up to $9.99

fraction with denominator of 100 to decimal

Addition of 4 digit nos.

Subtraction 5 digits with and without regrouping

Division by 1 digit(no remainder)

Ordering unit fractions

Identify parallel, intersecting and perpendicular lines

Identify improper fractions and mixed nos.

Convert improper fraction to mixed no.

Prime and composite numbers

Perimeter of polygons

GRADE 5 HOMEWORK ASSIGNMENT PLANNING

Homework Planning By Topic

Date	Topics of Lessons to be Taught	Current Topics			
	Unit 39 Addition of 3 or more addends and check	Add 3 or more addends & check			
	Unit 39 Solving word problems involving + of 3 or more addends	Add 3 or more addends & check			
	Unit 40 Subtract 6 digit no. & check		Subtract 6 digit # & check		
	Unit 40 Solving word problems involving subt. of 6 digit nos.		Subtract 6 digit # & check		
	Unit 41 Use pos. & neg. nos. in real life situations Use no. line to solve problems involving + and - nos.	Add 3 or more addends & check		Pos.& neg. nos.-real life situations	
	Unit 42 Solve problems by arranging info. in a list		Subtract 6 digit #	Use no. line to solve problems involving + and - nos.	Solve probl using a lis
	Unit 42 Solving problems by arranging info. in a table	Add 3 or more addends & check		Pos.& neg. nos.-real life situations	Solve problems using chart
	Unit 43 Identify prime nos. Identify composite nos.		Subtract 6 digit # (verbal)	Use no. line to solve problems involving + and - nos.	Solve problems using a lis
	Unit 43 Rename composite nos. as product of primes	Add 3 or more addends & check		Pos. & neg. nos.-real life situations	Solve probl using a cha
	Unit 44 Divide a dividend that is multiple of 1000 or 10,000 by a divisor that is a mult. of 100(comp.& verbal)		Subtract 6 digit #	Use no. line to solve problems involving + and - nos.	Solve probl using a lis
	Unit 44 Divide any 4 & 5 digit no. by divisor that is mult. of 100/computation	STUDY FOR TEST			

Review Topics

naming angles	mult. 2 digit by 2 digit	Per. of rect.	+ dec. (thru thous.)	x fract	LCM	
meas. angles	÷ 5 digit by 1 digit	Area of rect.	- dec. thru thous.	x mixed by whole		
naming angles	mult. 3 digit by 2 digit	÷ 5 digit by 2 digit		x mixed by mixed	GCF	
meas. angles		Per. of rect. (verbal)			LCM	
	÷ 5 digit by 1 digit			x mixed by mixed	GCF	
	mult. 2 digit by 2 digit	Area of rect. (verbal)	+ dec. thru thous.			
meas. & naming angles	÷ 5 digit by 2digit	Per.of rect. (verbal)		x mixed by whole		
Prime no.	÷ 5 digit by 1 digit	Area of rect. (verbal)	- dec. thru thous.			
Composite no. into prod. of primes	mult. 3 digit by 2 digit			x mixed by mixed	LCM	
Composite no. into prod. of primes	÷ where dividend is mult. of 1000 & divisor is mult. of 100	÷ 5 digit by 2 digit				

GRADE 5 POSSIBLE TEST TOPICS CORRELATED TO HOMEWORK PLANNING SHEET

Addition of 3 or more addends and check(compuation & verbal)

Subtraction of 6digit number and check(computation & verbal)

Positive and negative numbers--real life situations

Using number line to solve problems invovling pos. & neg. nos.

Prime and composite numbers.

Least Common Multiple

Greatest Common factor

Mult. fractions

Mult. mixed no. by whole no.

Mult. mixed no. by mixed number.

Perimeter, area of rectangle(computation and verbal)

Arranging information into list or chart to solve problems

Naming and measuring angles

Multiplication 2 digit no. by 2 digit no.

Multiplication 3 digit no. by 2 digit no.

Division of 5 digit number by 1 digit divisor and by 2-digit divisor.

Addition, subtraction decimals thru thousandths

GRADE 6

HOMEWORK ASSIGNMENT PLANNING

Homework Planning By Topic

ate	Topics of Lessons to be Taught	Current Topics					
	Unit 40 Defn. of integer Defn. of opposites	Defn. of opposites					
	Unit 40 Ordering of integers		Ordering integers				
	Unit 40 Verbal problems involving integers		ordering integers				
	Unit 41 Locate, read & interpret coord. on a graph	Defn. of opposites		read coord.			
	Unit 41 Graph points in all 4 quadrants		Ordering integers	read, graph coord.			
	Unit 42 Sum of 2 opposites equals 0 Sum of 2 pos. int.			graph points	Sum 2 opp. Sum 2 pos.		
	Unit 42 Sum of 2 neg. int. Sum pos. & neg. int.	Defn. of opposites		read coord.	Sum 2 neg. Sum 2 pos.		
	Unit 42 Verbal problems involving + of int.		Ordering integers	graph points	Sum pos.& neg. Sum 2 pos.	Add. int. (verbal)	
	Unit 43 Table of solutions for an equation with 2 variables			read coord.	Sum pos. & neg. Sum 2 neg.	Subt. int. (verbal)	Table solns. eqn. -2 variables
	Unit 43 Graph soln. set of eqn. in 2 var.	Defn. of opposites		graph points	Sum pos.& neg. Sum 2 pos.		Table solns. eqn.-2 variables
	Unit 44 4 digit x 4 digit		STUDY FOR TEST				

Review Topics

equiv. fract.	fract to ?/100	comparing dec.	% to dec.	circle graphs	+ dec	
whole - mixed			fract to %		2-step word problem	mode median
	fract to ?/100		dec to %	circle graphs	dec	
subt. unlike fract.			fract to %			median mean
equiv. fract.		comparing dec.			2-step word problem	
	fract to ?/100		dec to %	circle graphs	+ dec	
mixed - mixed (unlike den.)			% to dec		2-step word problem	
equiv. fract.			fract to %		- dec	
		comparing dec.		circle graphs		mean median mode
mixed - mixed (unlike den-verbal)					2-step word problem	

GRADE 6 POSSIBLE TEST TOPICS CORRELATED TO HOMEWORK PLANNING SHEET

Defn. of opposites
Ordering of integers(computation & verbal)

Reading and graphing points

Addition of integers(computation & verbal)

Volume rect. prism.

equivalent fractions.

subtration of fractions

subtraction mixed from whole no.

subtraction mixed no. from mixed no.

comparing decimals

% to decimals

Addition, subtraction decimals

decimals to %
fraction to %
fractions to ?/100

circle graph

Solving 2-step word problems

Mean, median, mode

Situations for Discussion by New Teachers: Homework

1. You discover, by looking over the first batch of homework papers, that one student writes fives and N's backwards and has generally very poor handwriting. What do you do?

2. While speaking to you about the several homework assignments that she has not done, a student explains that she can't do homework, since she works every afternoon and has heavy family responsibilities in the evening. What do you do?

TESTS

Questions to consider:

a. When should tests be given?

b. How much notice should students be given?

c. What is to be tested?

d. What types of questions should be given?

e. How long should a test be given?

f. How difficult should the test be?

g. What strategies should be given to the student to study for the test?

h. What should be looked at while making a test?

i. How do you use the test results?

j. . Should a test be reviewed in class after it is marked? When? How?

k. How does future homework relate to test results?

TESTING

1. **PLANNING OF TESTS:**

   a. Tests should be planned ahead. A worthwhile suggestion is to plan the next test as soon as the previous one is given after planning the homework assignments from one test to another.
   b. The teacher should begin by listing the topics to be tested as a result of the homework planning and classwork experiences.
   c. The teacher should consider the objectives of each question.

2. **SOURCE OF QUESTIONS:**

   a. Same type as homework.
   b. Same type as done in class.
   c. Use of other teacher-made tests from the past.
   d. Use of test booklets prepared by publishers.
   e. Use of previous state exams or school uniform exams.
   f. Use of texts.

3. **LENGTH OF TEST:**

   a. The teacher should take the test first, then multiply the time by about $5\frac{1}{2}$.

      Correct test length if necessary keeping in mind that time is needed for distribution and collection. The time factor by which one multiplies may vary depending upon the type of class that is being tested, the amount of reading involved with each question, etc.

4. **TYPE OF QUESTIONS:**

   a. short questions that are single-concept questions, multi-concept questions, verbally stated questions, etc.
   b. multiple choice questions
   c. long questions

5. **FORMAT OF TESTS:**

   a. The test should start off with easy questions, go to difficult and then to the more difficult questions i.e., ascending order of difficulty.
   b. Topics covered on last two days before the test should not be included on the test since students need to practice these topics through homework.
   c. Standards from State Education Department should be used in both the wording and the selection of test questions.
   d. Students should be informed as the point value on the test and the directions to be followed so that they may budget their time accordingly.

5. **FORMAT OF TESTS CONTINUED:**

e. Space should be allowed on tests for students to show work in an organized fashion.
f. Homework should be planned cyclically so that a frequent review of topics covered on the test may be done through the homework.
g. Tests should include "extra for experts" question(s) in which no actual points are given for correct work but an indication such as a "+" next to the grade. This "+" may be considered when determining a student's grade for the marking period or course.
g. Tests should be neat and not crowded. It is suggested that a draft of the actual test be written first, then changes made if necessary.
h. Duplication of the test should be done prior to the day of need so that the test may be taken by the teacher, proofread ahead of time so that students are not spending unnecessary time on a problem in which an error occurs, and a marking key may be made up as the proofreading is done.

6. **MARKING OF TESTS:**

a. Standards of marking as used in marking state exams should be used for marking of teacher-made tests.
b. A marking key should be used so that uniformity of grading takes place.
c. In marking a question where partial credit is to be given, the teacher should decide before marking how many credits are to be given to each step in the solution. If a problem contains several parts of which some are dependent on others, the teacher should not penalize the student several times but should give him credit if his work for one part is consistent with his work on the other part. i.e., if a student "follows through correctly", deduction is made only for the initial error made.
d. It is also suggested that if there are any long questions, the teacher should mark the same question on each paper first and then mark the second long question, etc. In this way the marking will be consistent on all student papers.
e. If an answer is wrong, put an x through the answer so that it cannot be changed without being noticed.
f. Besides putting the grade on the test, whenever possible try to write a comment that would encourage better performance or give credit to the students' work.
g. If comments concerning organization of students' work is warranted, include them either with the individual question or with the grade at the top of the test.
h. Try to have the tests marked by the day after it is given so that the greatest gains can occur in learning from mistakes which are "fresh in students minds". Do not go over a test unless the students have their papers in front on them.
i. As you mark the papers, keep a list of those students who you would want to write their solutions on the board when the test is gone over. (It is suggested that you have an alternate student in case of absence). One student may be chosen to do two or possibly three questions at the board depending upon the length of the questions.

j. An analysis of test results should be done so that you are aware of the type of errors that have been made so that emphasis can be made on the particulars in your future teaching.
k. It is NOT necessary to inform the students of the number of students failing, etc. (This type of discussion should be avoided with the students since it usually causes more problems than being helpful to the improvement of instruction).

7. **USE OF TEST RESULTS:**

a. An item analysis should be made. There should be a definite follow-up in which notes are made on the difficulty of each question.
b. The length of the test should be analyzed.
c. Comments should be made for future references.
d. An analysis of test results should be made for evaluation of success of various work taught. This analysis should make the teacher aware of what points to stress when the test is gone over.
e. Successive homework assignments should then include questions that caused more difficulty to students. This should be done through a spiral review of topics. These items may then be retested on the next test.
e. Students organization of work should be analyzed so that specific strategies can be given to students to improve their organization.
f. Common errors made by students should be determined to make changes in future teaching of these topics.

8. **RETURNING OF TESTS:**

a. Tests should be returned the very next day after it is given if possible. In this way it is fresh in the student's mind as to what he/she did on the test. It also shows the students that the teacher is concerned about the improvement of learning in a sequential subject.
b. Test papers should be arranged in order by rows to facilitate the distribution of the papers. The teacher, as he/she is marking the papers, should make a list of those students who should put up the work considering their work shown and not just the correct answer. If several approaches to the same problem appear, each method should be put up at the board. If no students get a particular question correct, the teacher should elicit the necessary steps in the solution by asking questions directed toward that goal.
c. A "Do Now" on the day of going over the test might be any of the following:
    a. Having students explain, in writing, definitions, rules or procedures for topics from the test.
    b. Having students explain, in writing, a comparison between the procedures needed for different topics from the test.
    c. Review of the pre-requisite skills needed for the next lesson.

The "Do Now" should not be similar problems to those causing difficulty on the test since if they couldn't do the problems and did not have another explanation of the solutions, they would not be able to do the "Do Now" exercises.

d. A discussion should take place with the students (prior to any distribution of test papers) on the purposes of going over a test. e.g. looking for one's point of error looking for alternative methods

   Students must feel that they have something to gain from this experience and that understanding what they got wrong has value for the future especially in our sequential subject matter.

   Sttudents must be informed as to the need for taking notes on problems whether they were right or wrong.

e. The board should be marked with the problem numbers and the students' names of those responsible for putting up the complete solution. Emphasis must be placed on the need for work shown that would be helpful to those students having difficulty with each problem. This assignment of students should have been done at the time of marking the papers.

f. Before distributing the papers, students should be told that if their name appears on the board that they are writing the work for the benefits of their classmates.

g. As students are writing on the board, the teacher should be walking around discussing individual papers with students, making suggestions to them, encouraging them individually, and checking that students are looking at the boardwork and taking notes.

h. Students are then to go to the board and explain (not just read) their work to their classmates. If questions arise, they should should be directed to the entire class for responses.

i. Strategies can be discussed with students on budgeting of time on a time, preventing careless errors, organizing work, techniques of studying for a math test, etc.

j. Students should be allowed to keep their test papers and stress should be placed on studying from them for future exams.

SUGGESTIONS FOR STUDENTS RELATED TO MATHEMATICS EXAMS

To the teacher: What additional suggestions would you give to students?

1. Start reviewing far enough in advance so you have time to do a careful, unhurried job.

2. Review by RE-DOING model problems, homework problems, class problems, problems from previous tests, and quizzes.

3. When you take a test, go in with an attitude that will help you. Have confidence in your own ability, but don't be over-confident. Don't try to "get by" with doing as little as possible. Don't worry to the point of anxiety. Fear alone can make a person do poorly on a test regardless of his ability and knowledge.

4. Follow directions. Read carefully. Consider the point value.

5. Look over the whole test quickly at the start and do the ones you are sure of first.

6. If you are unable to answer a question, leave it and go on to others, coming back to the hard ones later.

7. Be careful to show clearly what you are doing. Put in enough steps so that your method is perfectly clear.

8. Work neatly. It fosters accuracy.

9. Be organized in your solutions. It helps reduce the errors that could occur.

10. Check back for accuracy as you go along.

11. If you have extra time at the end of the test, re-check your work.

TAKING A MATH TEST

STRATEGIES THAT CAN HELP THOSE WHO CAN DO PROBLEMS IN CLASS BUT NOT ON THE TEST

1. Each day copy an assortment of problems on index cards. On the front of the card write the question, and on the reverse side write the complete solution. At least twice a week, shuffle the deck of index cards and deal yourself the number of problems you can complete in one hour. Work the problems and grade yourself by comparing your solutions with the ones found on the reverse side of the index cards. For each problem you fail to complete successfully, find at least two similar problems in your index cards, notes, or book. Complete and check these additional problems during the same reviewing session.
2. By placing your questions on index cards, you are providing a situation that closely resembles test conditions. That is, verbal hints from section titles are not easily accessible and the questions are in random order.
3. Re-doing rather than reading problems will show you whether you really know the mathematics.

STRATEGIES THAT CAN HELP THOSE WHO MAKES MANY CARELESS ERRORS ON A TEST.

1. Organization is important. It helps reduce the errors that could occur.
2. Work neatly. It fosters accuracy.
3. Check back for accuracy as you go along.
4. If you have extra time at the end of the test, re-check your work by re-doing each question.
5. Check your answers for reasonableness.
6. Re-read each question after you have completed your work to be certain you have answered the particular question.
7. Do not write over numbers. Rewrite numbers when changes occur.

STRATEGIES THAT CAN BE OF HELP TO THOSE STUDENTS WHO GET NERVOUS DURING A MATH TEST.

1. Start reviewing for enough in advance so you have time to do a careful, unhurried job.
2. Assume that the teacher will give a quiz tomorrow on what you've studied today.
3. When you take a test, go in with an attitude that will help you. Have confidence in your own ability, but don't be over-confident. Don't try to "get by" with doing as little as possible.
4. Don't worry to the point of anxiety. Fear alone can make a person do poorly on a test regardless of his ability and knowledge.
5. Do the questions that you are sure of first.
6. Get a good night's rest the night before an examination.

STRATEGIES THAT CAN HELP THE STUDENT WHO RUNS OUT OF TIME ON A TEST

1. Look over the whole test quickly at the start and do the ones you are sure of first.

2. Give careful thought to the first steps of the solution, write them out in detail at your normal pace. Mistakes made in the first few steps are usually penalized more heavily, since they may indicate a complete lack of knowledge on how to begin the problem.
3. Construct drawings that can improve your presentation of the problem and its solution. Record all formulas or theorems that seem to apply to the problem.
4. After completing two or three steps in the solution, quickly determine if you can complete the solution--in the limited time remaining--at a faster pace.
5. If you do not believe that enough time is available, outline the steps you would take to complete the problem without actually writing out the required statements, or computations. You are at least showing the teacher that you have enough knowledge to complete the problem.

CHECKLIST FOR EVALUATING A MATHEMATICS TEST

		Yes	NO
1.	Are there directions for taking the test?	___	___
2.	Are the directions simple and clear?	___	___
3.	Is the test legible?	___	___
4.	Is the test of appropriate difficulty?	___	___
5.	Is the test of appropriate length?	___	___
6.	Is space provided for showing work?	___	___
7.	Are answer spaces provided?	___	___
8.	Are point values indicated?	___	___
9.	Is duplication avoided? Is each concept tested only once?	___	___
10.	Do short questions test only one mathematical concept?	___	___
11.	Is there provision for writing extended responses?	___	___
12.	Is there provision for original problem solving as well as algorithmic exercises?	___	___
13.	Is there provision for extra credit for students who finish early?	___	___
14.	Is there a variety of types of problems on the test?	___	___
15.	Is there provision for reinforcing previously learned skills?	___	___
16.	Does the test include review items based on previous homework and class exercises?	___	___

Name: _____

Show all work. Questions 1-4 are 5 points each; questions 5-24 are 4 points each

1. $\begin{array}{r} 673 \\ +\ 281 \\ \hline \end{array}$

2. $\begin{array}{r} 367 \\ +\ 548 \\ \hline \end{array}$

3. $\begin{array}{r} 672 \\ -\ 91 \\ \hline \end{array}$

4. $\begin{array}{r} 403 \\ -\ 168 \\ \hline \end{array}$

5. $\begin{array}{r} 4.2 \\ +\ 3.7 \\ \hline \end{array}$

6. $\begin{array}{r} 8.9 \\ -\ 2.3 \\ \hline \end{array}$

7. $3\overline{)24}$

8. $2\overline{)14}$

9. $5\overline{)45}$

10. $4\overline{)16}$

11. $6\overline{)42}$

12. $\begin{array}{r} 3 \\ \times\ 7 \\ \hline \end{array}$

13. $\begin{array}{r} 5 \\ \times\ 5 \\ \hline \end{array}$

14. 8×2

15. 5×3

16. 7×6

17. $\begin{array}{r} 9 \\ \times\ 4 \\ \hline \end{array}$

18. $\frac{5}{8} \ ? \ \frac{1}{8}$

a. < b. > c. = d. not given

19. $\frac{1}{9} \ ? \ \frac{1}{6}$

a. < b. > c. = d. not given

20. $\frac{3}{8} + \frac{2}{8}$ equals

a. $\frac{5}{8}$ b. $\frac{1}{8}$ c. $\frac{5}{16}$ d. not given

21. $\frac{7}{10} - \frac{3}{10} =$

a. $\frac{4}{10}$ b. $\frac{10}{10}$ c. c. $\frac{4}{0}$ d. not given

22. $\frac{1}{3}$ of 12 equals

a. 24 b. 36 c. 8 d. not given

23. There are 363 students in the third grade and 439 in the fourth grade. What is the total number of students in these two grades?

a. 76 b. 802 c. 792 d. not given

24. Spin the spinner. Getting blue or green is:
a. not equally likely
b. equally likely
c. not given

GRADE 4 TEST NAME_____

All work must be shown . Questions 1-12 are 5 points each; questions 13-22 are 4 points each.

1. Which figure shows parallel lines?

a. b. c.

2. Which figure shows intersecting lines?

a. b. c.

3. Add: 2,773 + 8,608

4. Subtract: 53,639 - 23,728

5. Subtract: 48,524 - 26,712

6. $8.04 + $3.99

7. Change $\frac{14}{100}$ to a decimal

8. Identify the improper fraction:
a. $2\frac{4}{5}$ b. $\frac{5}{7}$ c. $\frac{12}{9}$ d. not given

9. Choose the greatest common factor of 16 and 20
a. 2 b. 8 c. 4 d. not given

10. The least common multiple for 6 and 8 is
a. 2 b. 24 c. 48 d. not given

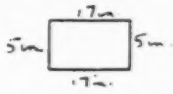
11. Use an integer to describe a gain of $8

12. Use an integer to describe 4 miles below sea level

13. $\frac{37}{4}$ is equal to
a. $8\frac{3}{4}$ b. $9\frac{1}{4}$ c. 9 d. not given

14. $4\overline{)480}$

15. Find the perimeter

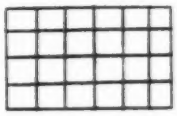


a. 22in.
b. 85 in.
c. 44 in.
d. not given

16. Which number is prime?
a. 4 b. 12 c. 7 d. 15

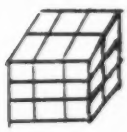
17. Which number is a composite number?
a. 23 b. 2 c. 27 d. 11

18. Find the area:



a. 20 sq. units
b. 24 sq. units
c. 26 sq. units
d. not given

19. Find the volume:



a. 24 cubic units
b. 8 cubic units
c. 14 cubic units
d. not given

20. How many outfits can be made from the following items?
brown pants, blue pants, white shirt, red shirt

21. $\frac{1}{5}$? $\frac{1}{8}$

a. < b. > c. = d. not given

22. In October, Cindy's arts and crafts shop sold $7,982 worth of goods. The shop took in $1,461 in November. How much money did the shop take in during this two-month period?

GRADE 5 TEST NAME_____

All work must be shown to receive credit. All questions are 5 points each except questions 13, 15, 16, 20 & 21 which are 4 points each.

(1) Add and check:

$$\begin{array}{r} 32{,}476 \\ +\ 20{,}761 \\ \hline \end{array}$$

(2) 16,145 + 179,443 + 180,007 =

(3) $\begin{array}{r} 183{,}762 \\ -\ 169{,}377 \\ \hline \end{array}$

(4) $\begin{array}{r} 81 \\ \times\ 42 \\ \hline \end{array}$

(5) $\begin{array}{r} 563 \\ \times\ 49 \\ \hline \end{array}$

(6) $6\overline{)16{,}386}$

(7) $25\overline{)17{,}050}$

(8) 812 ÷ 56 ÷ 8 ÷ 384 =

(9) $\begin{array}{r} 16.330 \\ -\ 5.732 \\ \hline \end{array}$

(10) 8.632 ÷ 0.738 =

(11) Find the Least Common Multiple of 3, 4, and 9

(12) Find the Greatest Common Factor of 24 and 40

(13) Which of these numbers are prime?
1, 2, 13, 21, 24

(14) Write 80 as a product of prime numbers.

(15-16) Use signed numbers to describe:

(15) 19 feet above sea level

(16) 12^{0} below zero

(17) $\frac{7}{8} \times \frac{3}{5}$ =

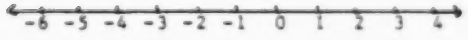
(18) $6\frac{1}{4} \times 6$ =

(19) $6\frac{5}{8} \times 2\frac{1}{3}$ =

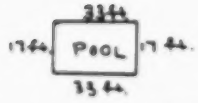
(20) Measure the angle below with your protractor

(21) Solve using the number line below:

A profit of $4 was followed by a loss of $5. What was the result?



(22) How much fencing is needed to surround the pool shown below?



(23) What is the area of the dining room shown below?

GRADE 6 TEST NAME_____

All work must be shown to receive credit. Questions 1-26 are 3 points each; Questions 27 & 28 are 5 points each; questions 29-34 are 2 points each.

(1) $63.94
8.36
+ 19.08

(2) $50.03
- 23.58

(3) 8.629 + 0.721 is equal to
(a) 7.908 (b) 9.340
(c) 9.450 (d) not given

(4) 3.5 ◯ 3.48
(a) < (b) >
(c) = (d) not given

(5) $\frac{3}{8} = \frac{?}{32}$

(6) $\frac{7}{16} - \frac{2}{16} = ?$

(7) $\frac{5}{8} - \frac{5}{16} = ?$

(8) $9 - 1\frac{5}{6}$

(9) $18\frac{1}{2} - 6\frac{1}{3}$

(10) $27\frac{1}{8} - 14\frac{3}{4}$

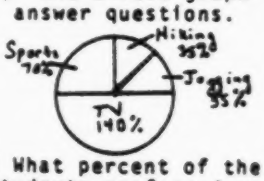
(11) $\frac{7}{100}$ as a percent is
(a) 0.7% (b) 7%
(c) 0.07% (d) not given

(12) .27 as a percent is
(a) 0.27% (b) 2.7%
(c) 27% (d) not given

(13) 30% as a decimal is
(a) 3.0 (b) 0.30
(c) 30.0 (d) not given

(14-15) $\frac{2}{5} = \frac{?}{100} = __\%$

(16-17) Use circle graph to answer questions.



(16) What percent of the students preferred sports for their entertainment?
(a) 70% (b) 25%
(c) 35% (d) not given

(17) Together what percent of the studen preferred sports or TV i r entertainment?
(a) 75% (b) 70%
(c) 210% (d) not given

(18) For 26, 27, 11, 33, 27, 21, and 17, find the median
(a) 21 (b) 26
(c) 27 (d) not given

(19) For 26, 27, 11, 33, 27, 21, and 17, find the mode
(a) 21 (b) 26
(c) 27 (d) not given

(20) For 75, 42, 65 and 78, find the mean to the nearest integer
(a) 86 (b) 70
(c) 65 (d) not given

(21) The opposite of -7 is
(a) $\frac{1}{7}$ (b) $-\frac{1}{7}$
(c) +7 (d) not given

(22) (+6) + (+9) equals
(a) -3 (b) +15
(c) +3 (d) not given

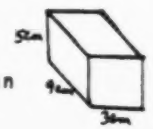
(23) (-6) + (-3) equals
(a) -3 (b) +9
(c) -9 (d) not given

(24) (+9) + (-8) equals
(a) +1 (b) -1
(c) +17 (d) not given

(25) -6 ? + 7
(a) < (b) >
(c) = (d) not given

(26) 0 ? -6
(a) < (b) >
(c) = (d) not given

(27) Find the volume

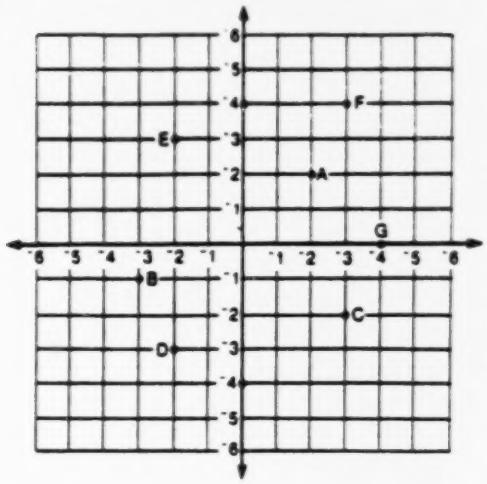


(a) 87 cm^3
(b) 135 cm^3
(c) 174 cm^3
(d) not given

(28) James bought 4 gifts that cost $7.16 each. How much change will he get from $50.00

see other side for (29-34)

Use graph to answer (29-34)



Give the ordered pairs for these points.

29. A _____ 30. B _____

31. C _____

Give the points for these ordered pairs.

32. (⁻2, ⁻3) _____ 33. (⁻4,0) _____

34. (⁻3, ⁻4) _____

Testing

Use the Checklist for Evaluating an Exam as a tool for evaluating the sample exam given in this manual.

- What items on the exam should be eliminated?
- How would you change the items on the exam?
- What additional items would you include?
- What are the advantages of providing point values?
- How would you improve the physical appearance of the exam?
- How could you provide for original problem solving on the exam?

Situations for Discussion by New Teachers: Testing

1. You give an exam on a unit of work and discover that 18 of your 30 students failed it. What do you do?

2. A student comes to you after class and complains that she understands the work, but simply can't do well on tests. What do you do?